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New Rudiments of Arithmetic

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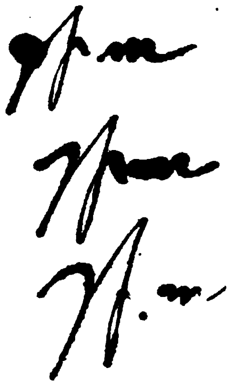

THOMSON'S NEW GRADED SERIES.

NEW

RUDIMENTS

OF

ARITHMETIC:

COMBINING

MENTAL AND SLATE EXERCISES

FOR

INTERMEDIATE DEPARTMENTS.

By JAMES B. THOMSON, LL. D.,

AUTHOR OF DAY & THOMSON'S ARITHMETICAL SERIES; EDITOR OF DAY'S SCHOOL ALGEBRA, LEGENDRE'S GEOMETRY, ETC.

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PREFACE.

THE "New Graded Series," of which this is the second book, is divided into three parts. The object of this arrangement is *convenience* and *economy.*

While there may be objections to an "indeterminate series of school-books," it must be admitted that exercises in reading, arithmetic, etc., which are adapted to the capacity of beginners, are totally unfit for advanced classes. In view of this fact, it requires no arguments to show that a "limited series," adapted to the different capacities of learners, is a dictate of common sense.

Each book in this Series is *complete* in itself. The definitions and principles, so far as each extends, are expressed in the *same language*, but the *examples* are all *different.*

The present work consists of a course of *Mental* and *Written Exercises combined.* It is designed:

1st. To develop the *elementary principles* of the science by *oral* examples.

2d. To familiarize the pupil with the *application* of these principles to the solution of problems requiring the use of the slate.

3d. To lead him to *generalize* the principles thus developed, and to put the steps of particular solutions into a *concise statement*, or *General Rule.*

4th. To secure *accuracy* and *rapidity* in the combination of numbers.

Finally, the work is specially adapted to intermediate classes, who are beginning to "cipher." The New Rudiments, it is hoped, may facilitate the progress of pupils, and merit the approval of teachers.

JAMES B. THOMSON.

NEW YORK, July, 1872.

SUGGESTIONS.

1. PARTICULAR attention should be paid to the assignment of Lessons. They should be neither too *long*, nor too *short;* but adapted to the *capacity* of the class, and the *time* they have for preparation.

2. *Thoroughness* should be insisted on, at every step. The acceptance of an imperfect lesson, whether from sympathy, or inattention, is a *positive injury* to the pupil.

3. The most effective auxiliaries of thoroughness are *frequent reviews* and *Tabular drills*. "Practice makes perfect."

4. A perfect recitation implies both *promptitude* and *correctness*. In reciting problems, the analysis should be *logical*, and the language *correct*.

5. Pupils should be encouraged to study out *different solutions* of the same problem, and to exercise their judgment in selecting the most *simple, logical*, and *concise*.

6. Care should also be taken to prevent the *habit of adding* by counting the fingers. *Counting* is not *addition*. Pupils should be taught to add numbers as a *whole*, and be able to name the *sum* of any two given digits, instantly.

7. The *definitions* should be carefully explained, and thoroughly committed to memory. Each principle and rule should be dwelt upon until the pupil comprehends it, and is able to give a *correct account* of it, in his own language, or that of the author.

8. Cultivate the *habit of self-reliance* in the solution of problems. It is better for the pupil to solve *one* example, *independent* of the answer and all extraneous aid, than a *dozen* by the help of a teacher, or a key.

9. Special pains should also be taken to cultivate the *perceptive faculties*, and correct the *erroneous ideas* of learners as to distance, surface, weight, etc.

10. In developing the idea of *Fractions*, and the *Units* of Weights and Measures, let the pupil *divide* some object into halves, thirds, etc., and, if possible, let him *see* and *handle* the *actual standards* of length, surface, capacity, and weight. These simple acts will give him a *more exact idea* of Fractions, and of Weights and Measures, than a score of pictures, or a talk an hour long.

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CONTENTS.

RUDIMENTS.

DEFINITIONS.

1. What is Arithmetic?

Arithmetic is the science of numbers.

2. What is a single thing called?

A ***Unit***, or ***One***.

3. If another is put with it, what?

Two.

4. If another, and another, etc., what?

Three, ***four***, ***five***, ***six***, etc.

5. What is number?

Number is a unit, or a collection of units.

6. When a number is not applied to any object, what is it called?

An ***Abstract Number***.

7. When it is applied to some object, what?

A ***Concrete Number***. Give examples.

8. When numbers express units of the *same kind*, as, 3 apples and 4 apples, 5 and 7, etc., what are they called?

Like Numbers.

9. When they express *units of different* kinds, as, 4 books and 6 pencils, what?

Unlike Numbers.

NOTATION.

10. What is Notation?

Notation is the art of expressing numbers by *figures, letters,* or *other numeral characters.*

11. What are the two principal methods in use?

The *Arabic* and the *Roman.*

ARABIC NOTATION.

1. What is the Arabic Notation, and why so called?

The ***Arabic Notation*** is the method of expressing numbers by *figures;* and is so called because it was introduced into Europe from Arabia.

2. How many figures does it employ, and what called?

Ten, called—

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

One, two, three, four, five, six, seven, eight, nine, naught.

3. What are the first nine called, and why?

The *first nine* are called *significant figures,* because each of them always expresses a *number.*

They are also called *digits,* from *digitus,* a finger, because the ancients used to reckon upon their fingers.

4. What is the last called, and why?

The *last* is called *naught,* because when standing alone it has *no value,* and when connected with significant figures, it denotes the *absence* of the orders in whose place it stands.

It is also called *zero,* or *cipher.*

Note.—The pupil should learn to distinguish and write the Arabic figures with readiness before proceeding further.

5. How is each of the first nine numbers expressed?

By a ***single figure.***

6. What are these numbers called?

Units of the ***first order,*** or simply *units.*

7. **What is the greatest number expressed by one figure?**

Nine.

8. **How is ten expressed?**

Ten is expressed by writing 1 in the *second place,* with a *cipher* on the right; as, 10.

9. **What is the 1 called, standing in the second place?**

A ***unit*** of the ***second order.***

1. Explain and write each of the numbers from ten to twenty.

Eleven is composed of *one ten* and *one unit,* and is expressed by writing 1 in the *second* place to denote the *ten,* and 1 in the *first* or right hand place to denote the *unit;* as, 11.

Twelve is composed of *one ten* and *two units,* and is expressed by writing 1 in the *second* place, and 2 in the *first;* as, 12.

Thirteen is composed of *one ten* and *three units,* and is expressed by writing 1 in the *second* place, and 3 in the *first;* as, 13, etc.

Twenty is *two tens,* and is expressed by placing 2 in the *second* place, and 0 in the *first;* as, 20.

2. Explain in like manner, and write each of the numbers from twenty to thirty.

3. From thirty to forty. From forty to fifty.

4. From fifty to sixty. From sixty to seventy; and so on to one hundred.

10. **What is the greatest number that can be expressed by *two* figures?**

Ninety-nine.

11. **How is a hundred expressed?**

A ***hundred*** is expressed by writing 1 in the *third* place, with *two ciphers* on the right; as, 100.

12. **What is the 1 called, standing in the third place?**

A ***unit*** of the ***third order.***

5. Explain and write each of the numbers from one hundred to one hundred and ten.

One hundred and one equals one hundred, no tens, and one unit, and is expressed by writing 1 in the *third* place, 0 in the *second*, and 1 in the *first*; as, 101.

One hundred and two is expressed by 102; one hun dred and three by 103, etc.

6. Explain and write each number from one hundred and ten to one hundred and twenty.

One hundred and ten is composed of one hundred, one ten, and no units, and is expressed by writing 1 in the *third* place, 1 in the *second*, and 0 in the *first*; as, 110.

One hundred and eleven by 111; one hundred and twelve by 112, etc.

7. Explain in like manner, and write the numbers from one hundred and twenty to one hundred and thirty.

8. Write the numbers from one hundred and thirty to one hundred and fifty. From one hundred and fifty to two hundred.

9. Write two hundred. Three hundred. Four hundred. Five hundred.

Write the following numbers in figures:

10. One hundred and twenty-three.
11. Two hundred and thirty-seven.
12. Three hundred and forty-five.
13. Four hundred and ten.
14. Six hundred and seven.
15. Five hundred and sixty-three.
16. Six hundred and five.
17. Seven hundred and thirty.
18. Six hundred and seventy-five.
19. Eight hundred and forty-three.
20. Nine hundred and ninety-nine.

13. What is the largest number that can be expressed by *three* figures?

Nine hundred and ninety-nine.

NOTE.—The preceding exercises should be repeated and supplemented by dictation, until the class become perfectly familiar with writing numbers less than a thousand.

14. How are numbers larger than 999 expressed?

By ***Other Orders,*** called *thousands, tens of thousands, hundreds of thousands, millions, tens of millions,* etc, each succeeding order having ***ten times*** the value of the preceding.

15. What is the general law by which the orders of units increase?

They increase from right to left by the scale of ten; that is,

Ten simple *units* make one *ten;*

Ten *tens* make one *hundred;*

Ten *hundreds* make one *thousand;* and, universally, *ten* of any *lower* order make *one* of the *next* higher.

16. What places do the different orders occupy?

Simple units occupy the *right hand* place;

Tens, the second place;

Hundreds, the third place;

Thousands, the fourth place;

Tens of thousands, the fifth place;

Hundreds of thousands, the sixth place;

Millions, the seventh place, etc.; the *order* of *units* corresponding with the place which the figure occupies.

17. What is the effect of moving a figure from right to left, or from left to right.

Its value is *increased tenfold* for every place it is moved from right to left; and is *diminished tenfold* for every place it is moved from left to right.

18. What are the different values of a figure called?

The *simple* and *local* values.

19. **What is the simple value of a figure?**

The ***simple value*** of a figure is the *number* of *units* it expresses when it stands alone.

20. **The local value of a figure? Illustrate both.**

The ***local value*** is the *number* it expresses when connected with other figures, and is determined by the place it occupies, counting from the right.

21. **What is the rule for expressing numbers by figures?**

Begin at the left hand, and write the figures of the given orders in their successive places toward the right.

If any intermediate orders are omitted, supply their places with ciphers.

Write the following numbers in figures:

20. One thousand, three hundred, and sixty.

21. Five thousand, seven hundred, and thirty-five.

22. Seven thousand, three hundred, and sixty-two.

23. Twenty-six thousand and seventy-five.

24. Thirty-seven thousand, one hundred, and six.

25. Ninety-five thousand and seventeen.

26. One hundred and twenty-three thousand and two hundred.

27. Three hundred and forty-eight thousand and two hundred.

28. Four hundred and ten thousand, three hundred, and forty.

29. Five hundred and forty thousand, six hundred, and thirty.

30. Six hundred thousand, two hundred and forty.

31. Seven hundred and fifty-five thousand, two hundred, and three.

32. Eight hundred and fifty thousand and three hundred.

33. Nine hundred and thirty-eight thousand and sixty-eight.

ROMAN NOTATION.

22. What is the Roman Notation; and why so called?

The ***Roman Notation*** is the method of expressing numbers by *letters;* and is so called because it was employed by the *Romans.*

23. How many, and what letters are used?

Seven capitals, viz.: I, V, X, L, C, D, and M.

24. What does each of these letters express?

The letter I expresses *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

TABLE.

I	denotes	one.	XXIV	denotes	twenty-four.
II	"	two.	XXV	"	twenty-five.
III	"	three.	XXVI	"	twenty-six.
IV	"	four.	XXVII	"	twenty-seven.
V	"	five.	XXVIII	"	twenty-eight.
VI	"	six.	XXIX	"	twenty-nine.
VII	"	seven.	XXX	"	thirty.
VIII	"	eight.	XL	"	forty.
IX	"	nine.	L	"	fifty.
X	"	ten.	LX	"	sixty.
XI	"	eleven.	LXX	"	seventy.
XII	"	twelve.	LXXX	"	eighty.
XIII	"	thirteen.	XC	"	ninety.
XIV	"	fourteen.	C	"	one hundred.
XV	"	fifteen.	CC	"	two hundred.
XVI	"	sixteen.	CCC	"	three hundred.
XVII	"	seventeen.	CCCC	"	four hundred.
XVIII	"	eighteen.	D	"	five hundred.
XIX	"	nineteen.	DC	"	six hundred.
XX	"	twenty.	DCC	"	seven hundred.
XXI	"	twenty-one.	DCCC,	"	eight hundred.
XXII	"	twenty-two.	DCCCC	"	nine hundred.
XXIII	"	twenty-three.	M	"	one thousand.

MDCCCLXXII, one thousand, eight hundred, and seventy-two.

NOTES.—1. Repeating a letter repeats its value. Thus, I denotes *one;* II, *two;* III, *three;* X, *ten;* XX, *twenty,* etc.

2. Placing a letter of *less* value *before* one of *greater* value, *diminishes* the value of the greater by that of the less; placing the less *after* the greater *increases* the value of the greater by that of the less. Thus, I denotes one, and V five; but IV is *four* and VI *six.*

3. Placing a *horizontal line* over a letter increases its value a *thousand* times. Thus, $\overline{\text{I}}$ denotes a thousand; $\overline{\text{X}}$, ten thousand; $\overline{\text{C}}$, a hundred thousand; $\overline{\text{M}}$, a million.

4. Four was formerly denoted by IIII; nine, by VIIII; forty, by XXXX; and ninety, by LXXXX.

Express the following numbers by letters:

1. 17.	7. 48.	13. 98.	19. 564.
2. 26.	8. 53.	14. 111.	20. 896.
3. 24.	9. 67.	15. 109.	21. 1116.
4. 37.	10. 78.	16. 114.	22. 1320.
5. 29.	11. 84.	17. 118.	23. 1536.
6. 44.	12. 89.	18. 377.	24. 1876.

NUMERATION.

25. What is Numeration?

Numeration is the art of reading numbers expressed by *figures, letters,* or *other numeral characters.*

26. How read numbers expressed by figures?

Divide them into periods of three figures each, counting from the right.

Beginning at the left hand, read the periods in succession, and add the name to each, except the last.

27. Why is the name of the last period omitted?

Because the *right hand* period always denotes *simple units,* therefore its name need not be mentioned.

Repeat the Numeration Table, beginning with units.

NUMERATION TABLE.

Hundreds of Trillions.	Tens of Trillions.	*Trillions.*	Hundreds of Billions.	Tens of Billions.	*Billions.*	Hundreds of Millions.	Tens of Millions.	*Millions.*	Hundreds of Thousands.	Tens of Thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*
2	9	8,	5	7	0,	9	2	3,	8	7	4,	2	6	7.
Period V. Trillions.			Period IV. Billions.			Period III. Millions.			Period II. Thousands.			Period I. Units.		

The periods in the Table are thus read : 298 trillions, 570 billions, 923 millions, 874 thousand, two hundred and sixty-seven.

NOTE.—This method of reading numbers is commonly ascribed to the *French*, and is thence called the *French Numeration*.

Others ascribe it to the *Italians*, and thence call it the *Italian Method*.

Copy and read the following:

1.	107.	13.	10315.	25.	3423201.
2.	260.	14.	12065.	26.	2300400.
3.	315.	15.	24308.	27.	5408025.
4.	809.	16.	13020.	28.	45320265.
5.	1020.	17.	20460.	29.	63205308.
6.	2405.	18.	35007.	30.	265310275.
7.	3200.	19.	40800.	31.	8045007.
8.	5007.	20.	85408.	32.	925604300.
9.	6080.	21.	115326.	33.	4260345809.
10.	7650.	22.	208065.	34.	61204400000.
11.	8075.	23.	400304.	35.	160240030045.
12.	9364.	24.	803025.	36.	407008300416.

Copy and read the following:

1. III.
2. VI.
3. IV.
4. VIII.
5. VII.
6. IX.
7. XIII.
8. XXXII.
9. XXVIII.
10. XLII.
11. LIX.
12. LVIII.
13. LXIX.
14. LIV.
15. LXXIII.
16. XLIX.
17. LXXXIV.
18. LXXXVIII.
19. XCV.
20. XCIX.
21. CIV.
22. CVIII.
23. CXII.
24. CIX.
25. CXL.
26. CLXXIX.
27. CCXC.
28. DXIX.
29. MDCCCXI.
30. MDCCCLXXV.

Express by figures, and read the following numbers:

1. Two hundred and five thousand, six hundred, and ninety-one.

2. Eight hundred and forty thousand, five hundred, and nine.

3. Two millions, four hundred thousand, and seventy.

4. Forty-five millions, sixty thousand, two hundred, and sixty.

5. Three hundred and ninety millions, four thousand, and seventy-two.

6. Six hundred millions, forty-eight thousand, and ten.

7. Five billions, six hundred and ten millions, and three hundred.

8. One hundred and forty billions, and thirty-five millions.

9. Forty-five millions, seven hundred and sixty thousand.

10. Three hundred and twenty-nine trillions, six hundred and thirty-seven billions, three hundred and forty millions, four hundred and nineteen thousand, two hundred and eighty-four.

NOTE.—Dictation exercises in reading and writing numbers should be continued, till the class is perfectly familiar with both.

ADDITION.

MENTAL EXERCISES.

To Teachers.—The object of this Exercise is to teach beginners the process of adding two digits together. If young, let them illustrate the examples by counters or unit marks.

1. If you have 1 apple, and I give you 1 apple more, how many apples will you have?

"One apple and 1 apple more are 2 apples."

2. If you have 2 cents, and you find 1 more, how many cents will you have?

"Two cents and 1 cent more are 3 cents."

3. How many are 3 marbles and 2 marbles?

4. Show this by your fingers.

5. Sarah has 3 red roses, and 3 white ones: how many roses has she of both kinds? Show it.

6. If an orange costs 5 cents, and a lemon 4 cents, how much will both cost? Show it.

7. William earned 6 cents in the morning, and 4 in the afternoon: how much did he earn in both?

8. Sanford obtained 6 credit marks, and his sister 6: how many did both obtain? Show it.

9. If I gather 6 quarts of cherries, and buy 8 quarts, how many quarts shall I have? Show it.

10. Joseph picked 5 quarts of blackberries, and his brother 7 quarts: how many quarts did both pick?

11. If I pay 8 dollars for a barrel of flour, and 7 dollars for a ton of coal, what shall I pay for both?

12. A teacher received two bouquets, one containing 8 flowers and the other 10: how many flowers were there in both?

13. George picked 12 peaches from one tree, and 3 from another: how many did he pick from both?

ADDITION TABLE.

2 and			3 and			4 and			5 and		
1	are	3	1	are	4	1	are	5	1	are	6
2	"	4	2	"	5	2	"	6	2	"	7
3	"	5	3	"	6	3	"	7	3	"	8
4	"	6	4	"	7	4	"	8	4	"	9
5	"	7	5	"	8	5	"	9	5	"	10
6	"	8	6	"	9	6	"	10	6	"	11
7	"	9	7	"	10	7	"	11	7	"	12
8	"	10	8	"	11	8	"	12	8	"	13
9	"	11	9	"	12	9	"	13	9	"	14
10	"	12	10	"	13	10	"	14	10	"	15
6 and			**7 and**			**8 and**			**9 and**		
1	are	7	1	are	8	1	are	9	1	are	10
2	"	8	2	"	9	2	"	10	2	"	11
3	"	9	3	"	10	3	"	11	3	"	12
4	"	10	4	"	11	4	"	12	4	"	13
5	"	11	5	"	12	5	"	13	5	"	14
6	"	12	6	"	13	6	"	14	6	"	15
7	"	13	7	"	14	7	"	15	7	"	16
8	"	14	8	"	15	8	"	16	8	"	17
9	"	15	9	"	16	9	"	17	9	"	18
10	"	16	10	"	17	10	"	18	10	"	19

1. Show by counters or unit marks how many 2 added to 3 will make.
2. Show how many 2 added to 4 will make.
3. Show how many 2 added to 5 will make.
4. Show how many 3 added to 4 will make.
5. Show how many 3 added to 5 will make.
6. Show how many 4 added to 5 will make, etc.

⁂ Particular care should be taken to see that beginners understand how the Addition Table is constructed; that they fully comprehend the results of adding two digits together, before they are required to commit them to memory.

DEFINITIONS.

1. What is Addition?

Addition is uniting two or more numbers in one.

2. What is the number obtained by addition called?

The ***Sum*** or *Amount*.

NOTE.—The sum or amount contains as many *units* or *ones* as all the numbers added.

3. When the numbers to be added are the same denomination, what is the operation called?

Simple Addition.

4. How is Addition denoted?

By a ***perpendicular cross*** called *plus* (+), placed between the numbers to be added. Thus, $5+3$ shows that 5 and 3 are to be added together, and is read, "5 plus 3," "5 and 3," or "5 added to 3."

NOTE.—The term *plus* is a Latin word, signifying *more*, or *added to*.

5. How is the equality between two numbers or sets of num bers denoted?

By ***two short parallel lines,*** called the *sign of equality* (=). The expression $5+3=8$, shows that 5 increased by 3 equals 8, and is read, "5 plus 3 equal 8," or the sum of "5 plus 3 equals 8."

Read the following expressions: $7+2=9$; $6+4=7+3$; $17+3+5=12+7+6$; $21+8+9=12+20+6$; $30+3+20=40+13$.

MENTAL EXERCISES.

TO TEACHERS.—The Mental and Slate Exercises are intended to be combined in each recitation. Hence, no more examples should be assigned to a lesson, than the class can thoroughly master.

1. Henry gave 4 cents for an orange, 3 cents for a pear, and 2 cents for an apple: how many cents did he give for all?

ANALYSIS.—4 cents and 3 cents are 7 cents, and 2 are 9 cents. Therefore, he gave 9 cents for all.

2. John paid 5 cents for a writing-book, and 2 cents for a pen: how many cents did he pay for both? 5 and what make 7?

3. If an orange costs 4 cents, a pear 3 cents, and a peach 2 cents, what will all three cost?

4. George gave 4 apricots to one of his sisters, 3 to another, and 5 to another: how many did he give to all?

5. If you pick up 4 apples under one tree, 3 under another, and 5 under another, how many apples will you have?

6. A man paid 3 dollars for a cane, 5 dollars for an umbrella, and 4 dollars for a hat: how much did he pay for all?

7. How many are 5 cents, and 3 cents, and 2 cents, and 1 cent?

8. Sarah paid 9 dollars for a hat, and 4 dollars for a parasol: what did she pay for both?

9. Count by *twos* rapidly to 60. Thus, two, four, six, eight, ten, twelve, etc.

10. Count by *threes* in like manner to 60.

11. Count by *fours* to 100.

SLATE EXERCISES.

Columns of Single Figures.

1. Write and add 2, 4, 5, 3, 6, and 1, upward and downward four times.

EXPLANATION.—Write the numbers in a column, and draw a line under it.

First. Beginning at the bottom, add upward; as, one, seven, ten, fifteen, nineteen, twenty-one. Set 21 under the column.

Second. Begin at the top, and add downward; as, two, six, eleven, etc.

$$\begin{array}{r} 2 \\ 4 \\ 5 \\ 3 \\ 6 \\ 1 \\ \hline 21 \end{array} \textit{ Ans.}$$

Copy and add the following upward and downward orally till rapidity is attained:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
2	3	4	5	2	3	4	5
4	2	3	2	1	2	1	3
3	4	5	3	5	4	3	2
1	2	1	2	2	3	4	5
2	1	3	3	1	1	2	2
2	2	2	1	3	2	5	4
1	3	4	2	2	3	3	3
2	2	3	3	4	4	3	4
3	1	2	2	1	2	4	5
4	5	5	4	5	3	5	4

*** Care should be taken to write figures with neatness and symmetry, and set them in perpendicular columns.

MENTAL EXERCISES.

1. There are 5 ducks in one pond, 6 in another, and 7 in another: how many are there in all the ponds?

2. A man picked 8 quarts of blackberries, and his son 6 quarts: how many quarts did both pick?

3. William has 9 chickens in one coop, and 7 in another: how many chickens has he in both?

4. 7 and what make 16? 9 from 16 leaves what?

5. How many are 7, and 6, and 5, and 4, and 8?

6. If there are 9 pears in one dish, and 8 in another, how many are there in both dishes?

7. 8 and what make 17? 9 from 17 leaves what?

8. Henry solved 6 examples, George 9, and Samuel 8: how many did they all solve?

9. Lucy gave her teacher 7 roses, Julia 9, and Hattie 10: how many roses had the teacher?

10. Count by fives to 100, with rapidity.

11. Count by sixes to 100, in like manner.

SLATE EXERCISES.

Copy and add the following as before:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
6	5	4	7	8	7	4	9
3	4	5	4	6	5	7	5
5	2	6	7	2	8	9	4
4	5	7	3	8	3	1	9
2	3	4	5	4	4	3	6
5	4	7	7	8	8	9	9
4	6	3	4	5	6	7	8
6	4	7	8	9	5	6	4
3	7	6	9	4	8	4	3
7	5	3	5	6	7	8	9

MENTAL EXERCISES.

1. Henry had 5 cents, and earned 6 more: how many cents had he then? 5 and what make 11?

2. William picked 7 quarts of cherries, and his brother 6 quarts: how many quarts did both pick?

3. 5 and what make 12? 6 and what make 13?

4. A farmer had 8 cows, and bought 6 more: how many had he then?

5. If you pay 7 dollars for a velocipede, and 8 dollars for an overcoat, what will both cost you?

6. 6 and what make 14? 7 and what make 15?

7. If Helen pays 8 dollars for a music-box, and 9 dollars for a fur cape, what will both cost her?

8. How many are 9 dollars, and 6 dollars, and 7 dollars?

9. How many are 15 rods, 6 rods, and 8 rods?

10. Count by sevens to 100, with rapidity.

11. Count by eights to 100, in like manner.

SLATE EXERCISES.

Copy and add the following upward and downward as before:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
4	3	2	9	5	6	8	9
3	4	3	8	3	8	8	8
6	2	5	4	8	9	5	4
8	3	7	3	4	7	6	7
1	1	6	1	2	4	7	5
2	6	1	7	3	5	3	6
7	7	3	6	9	3	5	7
3	2	4	5	6	7	9	8
5	7	6	8	7	6	5	7
8	6	7	4	6	7	8	9

Write in columns, and add the following:

9. 3 pounds, 4 pounds, 5 pounds, 2 pounds, 7 pounds, and 1 pound.

10. 6 yards, 5 yards, 3 yards, 4 yards, 2 yards, 8 yards, and 7 yards.

MENTAL EXERCISES.

1. How many are 3 and 10? 13 and 10? 23 and 10? 33 and 10? 43 and 10? 53 and 10? 63 and 10? 73 and 10? 83 and 10? 93 and 10?

2. 7 and 10? 17 and 10? 27 and 10? 37 and 10? 47 and 10? 57 and 10? 67 and 10? 77 and 10? 87 and 10? 97 and 10?

3. 3 and 4? 13 and 4? 23 and 4? 33 and 4? 53 and 4? 43 and 4? 63 and 4? 83 and 4? 93 and 4?

4. 5 and 3? 15 and 3? 25 and 3? 45 and 3? 35 and 3? 55 and 3? 65 and 3? 85 and 3? 75 and 3? 95 and 3?

5. 17 and 5? 37 and 5? 27 and 5? 57 and 5? 47 and 5? 67 and 5? 87 and 5? 77 and 5? 97 and 5?

6. 15 and 6? 35 and 6? 25 and 6? 45 and 6? 65 and 6? 55 and 6? 75 and 6? 95 and 6? 85 and 6?

7. 18 and 7? 28 and 7? 38 and 7? 48 and 7? 58 and 7? 68 and 7? 78 and 7? 88 and 7? 98 and 7?

8. 16 and 8? 26 and 8? 36 and 8? 46 and 8? 56 and 8? 66 and 8? 76 and 8? 86 and 8? 96 and 8?

9. 14 and 9? 24 and 9? 34 and 9? 44 and 9? 54 and 9? 64 and 9? 74 and 9? 84 and 9? 94 and 9?

10. Count by nines to 100, with rapidity.

11. Count by tens to 100, in like manner.

SLATE EXERCISES.

When the Sum of a Column is *Less* than 10.

1. What is the sum of 234 dollars, 423 dollars, and 132 dollars?

Operation.

$$\begin{array}{r} 234 \\ 423 \\ 132 \\ \hline 789 \text{ dols.} \end{array}$$

Analysis.—Write the numbers one under another, the units figures in one column, the tens in the next, and so on. Begin at the right and add: 2 units and 3 units are 5 units, and 4 are 9 units. Set the 9 under the units column, because it is *units*. Next, 3 tens and 2 tens are 5 tens, and 3 are 8 tens. Set the 8 under the tens column, because it is *tens*. Finally, 1 hundred and 4 hundred are 5 hundred, and 2 are 7 hundred. Set the 7 under the hundreds, because it is *hundreds*.

Note.—In practice, it is better simply to pronounce the results; as, two, five, nine, etc.

Copy and add the following, in like manner:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
21	32	14	43	32	321	124	434
34	11	23	21	20	132	531	243
12	45	42	34	45	434	233	322

Write in columns, and add the following:

10. $25+12+30$.	13. $34+21+40$.
11. $31+22+24$.	14. $46+30+12$.
12. $40+13+36$.	15. $51+25+23$.

6. How do you write numbers to be added?

Write units under units, tens under tens, etc.

7. Where begin to add, and how proceed?

Begin at the right, and add each column separately.

8. When the sum of a column is less than 10, what is done with it; and why?

Set it under the column added; because it is the *same order* as that column.

9. What two principles are necessary to be observed in addition?

1st. The numbers must be ***Like numbers.***

2d. ***Units*** of the ***same order*** must be added, each to each.

Can 3 books be added to 5 pencils? Why?

Do 4 units and 3 tens make 7 units, or 7 tens? Why?

MENTAL EXERCISES.

1. A certain school had 40 girls and 30 boys in attendance: how many pupils did it contain?

ANALYSIS.—40 is equal to 4 tens, and 30 is equal to 3 tens. Now 4 tens and 3 tens are 7 tens, or 70. The school, therefore, contained 70 pupils.

2. 50 is how many tens? 40? 60? 70? 80? 100?
3. 5 tens are how many? 7 tens? 6 tens? 9 tens? 8 tens? 10 tens?
4. How many are 3 tens and 5 tens? 6 tens and 4 tens? 5 tens and 7 tens?
5. How many are 20 and 30? 50 and 80?
6. How many are 70 and 50? 80 and 90?

7. In a certain grove there are 89 sugar-maples and 46 elms: how many trees are there in the grove?

ANALYSIS.—89 is 8 tens and 9 units, and 46 is 4 tens and 6 units. Now 8 tens and 4 tens are 12 tens, or 120; and 9 units and 6 units are 15 units, which, added to 120, make 135. Therefore, there are 135 trees in the grove.

NOTE.—When numbers to be added *mentally* are large, it is advisable to separate them into the units, tens, etc., of which they are composed, and begin to add with the highest order.

8. How many are 34 and 43? 26 and 51?

9. How many are 45 and 62? 71 and 46?

10. A man paid 68 dollars for a cow, and 55 dollars for a colt: what did he pay for both?

SLATE EXERCISES.

When the Sum of a Column is 10, or More.

1. A farmer had 3 flocks of sheep; one of 325, another 436, the other 541: how many sheep had he?

OPERATION.
325 s.
436 s.
541 s.
Ans. 1302 s.

ANALYSIS.—We write units under units, tens under tens, etc., and begin at the right hand as before. Thus, 1 unit and 6 units are 7 units, and 5 are 12 units, or 1 ten and 2 units. We set the 2 units under the column added, because it is the *same order* as this column, and add the 1 ten to the next column, because it is the *same order* as that column. Now 1 ten added to 4 tens makes 5 tens, and 3 are 8 tens, and 2 are 10 tens, or 1 hundred and 0 tens. We set the 0, or right hand figure, under the column added, because *there are no units* of this order, and add the 1 hundred to the next column, because it is the *same order* as that column. Adding the 1 hundred to the next column, the sum is 13 hundred, or 1 thousand and 3 hundred. This being the last column, we set down the *whole sum*, putting the 3 under the column added, because it is the *same order* as this column, and the 1 in the next, or thousands place, because it is *thousands*. Therefore, etc.

10. When the sum of a column *exceeds* 9, what do you do?

Write the units figure under the column, and add the tens to the next higher order.

11. What do you do with the last column?

Set down the *whole sum.*

12. What is adding the tens to the next order called?

Carrying the tens.

Carrying Illustrated by Unit Marks.

<table>
<tr><th>Thou.</th><th>Hund.</th><th>Tens.</th><th>Units.</th></tr>
<tr><td></td><td>|||</td><td>||</td><td>|||||</td></tr>
<tr><td></td><td>||||</td><td>|||</td><td>||||||</td></tr>
<tr><td></td><td>|||||</td><td>||||</td><td>|</td></tr>
<tr><td>M.</td><td>H.</td><td>T.</td><td></td></tr>
<tr><td>Am't |</td><td>|||</td><td>0</td><td>||</td></tr>
</table>

ANALYSIS.—325 = 3 hundred + 2 tens + 5 units; 436 = 4 hundred + 3 tens + 6 units; and 541 = 5 hundred + 4 tens + 1 unit. Now, to represent the first number, we place *three* counters in the column of hundreds, *two* in the column of tens, and *five* in the column of units. The other numbers are represented in a similar manner. Beginning at the right hand, we find there are 12 counters in units column. Withdrawing *ten* of them, *two* will be left, which we put under the column added. Now 10 units make 1 ten; hence, to represent the *ten* units withdrawn, we put a single counter (T.), denoting a unit of the 2d order, in the column of tens. Again, adding the column of tens, we find there are ten counters, and withdrawing *ten* tens from this number, no tens are left. To express the *absence* of tens, we put a *cipher* in tens place. But *ten* tens make 1 hundred; hence, to represent the *ten* tens withdrawn, we put a single counter (H.), denoting a unit of the 3d order, in the column of hundreds. Finally, in the hundreds column, there are 13 counters, and withdrawing *ten* of them leaves *three.* We therefore put *three* counters in the column of hundreds, and to represent the *ten* withdrawn, we put a single counter (M.), denoting a unit of the 4th order, in the column of thousands. The amount is 1 thousand, 3 hundred, 0 tens, and 2 units. Hence, *carrying the tens* is simply taking a part from a *lower* order and adding it to the *next higher,* which can no more affect the *amount* than it will affect the amount of money a man has, if he changes 10 cents for a dime.

(2.)	(3.)	(4.)	(5.)	(6.)
4683	3605	5072	7304	8097
3427	7036	3816	6084	3845
5032	5783	7301	3752	5432

13. The preceding principles may be summed up in the following

GENERAL RULE.

I. *Place the numbers one under another, units under units, etc., and beginning at the right, add each column separately.*

II. *If the sum of a column does not exceed* NINE, *write it under the column added.*

If the sum exceeds NINE, *write the units figure under the column, and carry the tens to the next higher order.*

Finally, set down the whole sum of the last column.

PROOF.—*Begin at the top and add each column downward. If the two results agree, the work is right.*

NOTE.—This method of proof depends upon the principle that *reversing* the *order* of the figures will be likely to *detect* any error that may have occurred in the operation. The learner should prove every answer.

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)	(5.)
Yards.	Pounds.	Rods.	Dollars.	Acres.
135	333	496	542	604
263	664	175	37	160
786	548	586	764	489
182	345	257	343	853
348	563	845	577	348

(6.)	(7.)	(8.)	(9.)	(10.)
684	103	496	840	965
937	85	37	6	4
685	967	4	28	382
129	49	132	394	41
845	732	563	825	985

11. Herbert read 235 pages one day, 264 the next, and 362 the next: how many pages did he read in all?

12. What is the sum of 2362 days, 375 days, and 27 days?

13. If a yoke of oxen cost 250 dollars, and a cart 119 dollars, what will both cost?

14. A man paid 67 dollars for his coat, 16 for his vest, 23 for his pants, and 13 dollars for his boots: what did he pay for his suit?

15. A farmer has four flocks of sheep, one flock containing 256 sheep, the second 320, the third 195, and the fourth 168: how many sheep had he?

16. What is the sum of five hundred and sixty-one, two hundred and seven, and nine hundred and fourteen?

17. What is the sum of twelve thousand and twelve, six thousand and two, and ninety-five hundred?

18. A man paid 2250 dollars for his farm, 1600 dollars for stock, and had 168 dollars left: how much money had he at first?

19. Washington was born in the year 1732, and lived 67 years: what year did he die?

20. A man paid 6270 dollars for a horse and sold it for 1565 dollars more than he paid for it: how much did he get for it?

21. In what year will a person born in 1865, be 21 years old?

22. What is the sum of 643 yards, 820 yards, 605 yards, and 319 yards?

23. If I pay 925 dollars for house rent, 430 dollars for clothing, and 768 dollars for other expenses, how much shall I spend in a year?

24. The age of four brothers is 89, 84, 78, and 67 years respectively: what is their united age?

25. John has 63 marbles, Henry 41, and William as many as both the others: how many did they all have?

26. What is the sum of 453 dols., 269 dols., 804 dols., 1000 dols.?

27. In a certain army there are 28260 infantry, 16325 cavalry, and 1328 artillery: how many men did the army contain?

28. A man bequeathed his wife 23260 dols., his son 17380 dols., and his daughter the same as his son: how much did he leave them all?

29. What is the sum of 365 days, 873 days, 219 days, and 35 days?

30. If a vessel sails 235 miles a day on three successive days, how far will she be from port?

31. My neighbor's farm contains 563 acres, and my own 435 acres: how many acres do they both contain?

32. Required the sum of 1725 years + 1007 years + 8520 years.

33. Required the sum of 1308 ounces + 710 ounces + 353 ounces and 42 ounces.

34. Required the sum of 2103 pounds + 106 pounds + 26 pounds + 89 pounds + 645 pounds.

35. If a man annually receives 1350 dollars salary, and 350 dollars interest, what is his annual income?

36. A man has four farms; one containing 340 acres, another 235 acres, another 250 acres, and the other 178 acres: how many acres were there in all?

(37.)	(38.)	(39.)	(40.)		(41.)		(42.)	
Dols.	Dols.	Dols.	Dols.	Cts.	Dols.	Cts.	Dols.	Cts.
35	82	423	3	45	4	26	5	75
42	40	607	2	76	3	75	3	81
37	61	440	6	08	4	90	1	09
61	43	851	4	30	5	71	6	75
70	17	760	7	05	2	43	2	33
25	28	978	8	26	3	78	8	45
38	73	465	7	40	9	25	9	67
47	68	886	2	61	4	08	7	30
63	94	529	8	35	6	25	8	05
85	87	735	3	42	4	16	9	35

(43.)		(44.)		(45.)		(46.)		(47.)	
Dols.	Cts.	Dols.	Cts.	Dols.	Cts.	Dols.	Cts.	Dols.	Cts.
29	13	40	53	49	31	52	60	467	53
30	50	24	47	12	53	29	35	613	64
42	35	63	15	64	31	42	18	87	02
10	78	70	40	52	49	60	20	751	34
26	42	34	85	38	24	73	00	872	60
72	96	62	12	63	19	28	67	493	04
26	04	73	68	72	43	49	28	81	73
50	30	58	76	30	04	83	00	757	02
29	17	82	94	72	85	16	27	563	40
36	23	64	47	37	23	40	23	80	33
47	58	87	28	52	92	71	19	1	94
53	22	92	86	46	25	83	24	693	03
84	35	73	52	56	83	85	36	903	48

*** **Exercises in adding long columns upon the slate or black-board are highly useful in acquiring accuracy and rapidity, and should be supplemented by the teacher. Columns of single figures are preferable for beginners.**

SUBTRACTION.

MENTAL EXERCISES.

To Teachers.—The design of this Exercise is to teach the pupil how to illustrate the *process* and the *result* of taking one number from another.

1. If you have 2 apples, and a boy takes 1 of them away, how many will you have left?

"One apple taken from 2 apples leaves 1 apple."

2. Show this by your fingers or unit marks.

3. If you have 3 peaches, and give 1 of them to your sister, how many will you then have? Show this.

4. If you have 5 cents, and lose 2 of them, how many will you have? Show this.

5. George had 4 pears, and sold 2 of them: how many did he then have? Show this.

6. Jennie had 6 roses, and gave 3 of them to her teacher: how many had she left? Show this.

7. James had 6 cents, and spent 4 of them for candy: how many cents did he have left? Show this.

8. A schoolboy had 10 marbles, and lost 5 of them: how many had he left? Show this.

9. If you buy a slate for 8 cents, and sell it for 5 cents, how much will you lose? Show this.

10. William gained 7 credit-marks, but lost 3 by bad conduct: how many did he then have?

11. The price of a hat is 6 dollars, and that of a cap 2 dollars: what is the difference in their price?

12. In a certain class there are 9 girls and 6 boys: how many more girls than boys in the class?

13. If you pay 5 cents for an orange, and sell it for 10 cents, how much will you gain?

14. 4 from 10 leaves how many? 4 from 12?

SUBTRACTION TABLE.

2 from			3 from			4 from			5 from		
2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
3	“	1	4	“	1	5	“	1	6	“	1
4	“	2	5	“	2	6	“	2	7	“	2
5	“	3	6	“	3	7	“	3	8	“	3
6	“	4	7	“	4	8	“	4	9	“	4
7	“	5	8	“	5	9	“	5	10	“	5
8	“	6	9	“	6	10	“	6	11	“	6
9	“	7	10	“	7	11	“	7	12	“	7
10	“	8	11	“	8	12	“	8	13	“	8
11	“	9	12	“	9	13	“	9	14	“	9
12	“	10	13	“	10	14	“	10	15	“	10

6 from			7 from			8 from			9 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0
7	“	1	8	“	1	9	“	1	10	“	1
8	“	2	9	“	2	10	“	2	11	“	2
9	“	3	10	“	3	11	“	3	12	“	3
10	“	4	11	“	4	12	“	4	13	“	4
11	“	5	12	“	5	13	“	5	14	“	5
12	“	6	13	“	6	14	“	6	15	“	6
13	“	7	14	“	7	15	“	7	16	“	7
14	“	8	15	“	8	16	“	8	17	“	8
15	“	9	16	“	9	17	“	9	18	“	9
16	“	10	17	“	10	18	“	10	19	“	10

1. Show by counters or unit marks how many 2 taken from 3 will leave?
2. Show how many 2 taken from 4 will leave.
3. Show how many 2 taken from 5 will leave.
4. Show how many 3 taken from 7 will leave.
5. Show how many 3 taken from 8 will leave, etc.

NOTE.—It is advisable to let young pupils *verify* the *results* of the Subtraction Table by counters or unit marks, before they are required to commit it to memory.

DEFINITIONS.

1. What is Subtraction?

Subtraction is *taking one number from another.*

2. What is the number to be subtracted called?

The ***Subtrahend.***

3. The number from which the subtraction is made?

The ***Minuend.***

4. What is the number obtained by subtraction called?

The ***Difference,*** or *remainder.*

1. When it is said that 5 taken from 9 leaves 4, which is the minuend? The subtrahend? The remainder?

2. When it is said that 6 taken from 14 leaves 8, what is the 6 called? The 14? The 8?

5. When both numbers are the *same denomination,* what is the operation called?

Simple Subtraction.

6. How is Subtraction denoted?

By a ***short horizontal line,*** called *minus* (–). When placed between two numbers, this sign shows that the number *after* it is to be taken from the one *before* it. Thus, 6 – 4, shows that 4 is to be taken from 6, and is read "6 minus 4," or "6 less 4."

NOTE.—The term *minus* is a Latin word, signifying *less.*

Read the following expressions:

1. $12 - 5 = 14 - 7$.
2. $15 - 3 = 10 + 2$.
3. $35 - 10 = 30 - 5$.
4. $20 - 6 = 8 + 6$.
5. $50 - 12 = 30 + 8$.
6. $75 + 25 = 105 - 10 + 5$.

MENTAL EXERCISES.

1. If you pay 9 cents for a sponge, and sell it for 5 cents, how much will you lose?

ANALYSIS.—Five cents from 9 cents leave 4 cents. Therefore, you will lose 4 cents.

2. A farmer having 10 cows, sold 4 of them: how many had he left? 6 and what make ten?

3. Homer is 11 years old, and his sister 5 years: what is the difference in their ages?

4. Susan had 8 pinks, and gave 3 to one of her schoolmates: how many had she left? 3 and what make 8?

5. If you have 9 doves, and 3 of them fly away, how many will you have left?

6. George having 12 dollars, gave 4 dollars for a pair of skates: how much had he left?

7. The price of a vest is 8 dollars, and that of a coat 14 dollars: what is the difference in the price?

8. A farmer had 13 cows, and sold 4 of them: how many did he then have? 4 and what make 13?

9. Henry is 11 years old, and Charles 6 years: what is the difference in their ages? 6 and what make 11?

10. Count backward by *twos* from 50 to 0, with rapidity. Thus, fifty, forty-eight, forty-six, forty-four, etc.

11. Count backward by *threes* from 60 to 0.

12. Count backward by *fours* from 60 to 0.

SLATE EXERCISES.

Copy the following, and subtracting the lower number from the upper, set the result under the figure subtracted. Repeat the operation till it can be performed without hesitation.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From	7	8	7	9	8	8	6	9
Take	4	5	6	5	6	7	6	5

	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
From	8	7	11	9	8	10	14	9
Take	6	5	7	7	8	7	9	8

MENTAL EXERCISES.

1. John bought 10 peaches, and gave 6 of them to his brother: how many did he have left? 6 and what make 10?

2. A man had 16 horses, and sold 7 of them: how many had he left? 7 and what make 16?

3. Julia solved 17 examples, and Harriet 8 examples: how many more did Julia solve than Harriet?

4. 8 from 11 leaves how many? 8 from 16? 8 from 14? 8 from 15?

5. If a man earns 15 dollars a month, and spends 9 dollars, how many dollars will he have left?

6. 9 from 15 leaves how many? 9 from 17? 9 from 14?

7. A market-boy had 18 eggs in his basket, and letting it fall broke 8 of them: how many whole ones did he have left?

8. Frank having 8 cents, wishes to buy a slate which costs 12 cents: how many cents more does he need, to pay for the slate?

9. Count backward by *fives* from 70 to 0 with rapidity.

10. Count backward by *sixes* from 72 to 0, in like manner.

SLATE EXERCISES.

Copy and subtract the following as above:

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
From	9	11	10	12	14	13	16	17
Take	6	7	8	7	9	8	7	9

	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
From	14	16	15	13	12	16	17	19
Take	6	7	8	5	8	7	8	9

MENTAL EXERCISES.

1. George had 12 apples, and gave 5 of them away: how many had he left? 12 less 7 are how many?

2. Horace is 14 years old, and his sister is 9: what is the difference in their ages? 14 less 5 are how many?

3. A person having 20 acres of land, sold 10 acres: how much did he then have?

4. 18 less 8 are how many? 17 less 5? 15 less 7?

5. If from a piece of silk containing 19 yards, 10 yards are cut, how much will be left?

6. What is the difference between 17 dols. and 9 dols.?

7. If you pay 17 dollars for a goat and sell it for 9 dols., what will be your loss?

8. If you buy a calf for 8 dollars, and sell it for 14 dols., what will be your gain?

9. A gardener set out 18 peach trees, and 9 of them died: how many lived?

10. Count backward by *sevens* from 70 to 0, as before.

11. Count backward by *eights* from 80 to 0.

SLATE EXERCISES.

When each Figure in the Lower Number is *Less* than the one above it.

1. What is the difference between 465 dollars and 123 dollars?

OPERATION.	
465	dols.
123	dols.
342	dols.

ANALYSIS.—Write the *less* number under the *greater*, placing the *units* under *units*, the *tens* under *tens*, and the *hundreds* under *hundreds*. Begin at the right, and proceed thus: 3 units from 5 units leave 2 units. Set the 2 in units place, under the figure subtracted, because it is *units*. Next, 2 tens from 6 tens leave 4 tens. Set the 4 in tens place, under the figure subtracted, because it is *tens*. Finally, 1 hundred from 4 hundreds leaves 3 hundreds. Set the 3 under the hundreds column because it is *hundreds*. The difference is 342 dollars.

Solve the following examples in the same manner:

	(2.)	(3.)	(4.)	(5.)	(6.)
From	629	745	846	4382	7468
Take	416	421	526	2150	3405

7. How do you write numbers for subtraction?

Write the less number under the greater, units under units, tens under tens, etc.

8. Where do you begin to subtract, and where put the result?

Begin at the right hand, and set the result under the figure subtracted.

9. What two principles are necessary to be observed in subtraction?

1st. The numbers must be ***Like numbers.***

2d. ***Units*** of the ***same order*** must be subtracted, one from the other.

Can 3 pears be taken from 5 inkstands?

Explain the reason.

Do 3 units from 7 tens leave 4 units, or 4 tens?

MENTAL EXERCISES.

1. 10 from 16 leaves how many? 10 from 26? 10 from 46? 10 from 76? 10 from 86? 10 from 96?

2. 10 from 27? 10 from 37? 10 from 57? 10 from 47? 10 from 67? 10 from 87? 10 from 77? 10 from 97?

3. 10 from 24? From 35? From 48? From 57? From 63? From 76? From 83? From 92?

4. Take 4 from 7. 4 from 17. 4 from 27. 4 from 57. 4 from 47. 4 from 37. 4 from 67. 4 from 87. 4 from 97. 4 from 77.

5. Take 5 from 8. 5 from 18. 5 from 28. 5 from 48. 5 from 38. 5 from 58. 5 from 78. 5 from 68. 5 from 88. 5 from 98.

6. Take 6 from 19. 6 from 29. 6 from 69. 6 from 49. 6 from 59. 6 from 79. 6 from 99. 6 from 89.

7. Take 7 from 16. 7 from 26. 7 from 46. 7 from 36. 7 from 56. 7 from 76. 7 from 66. 7 from 86. 7 from 96.

8. 2 from 11. 2 from 21. 2 from 31. 2 from 41. 2 from 51. 2 from 61. 2 from 71. 2 from 81. 2 from 91.

9. 4 from 22. 4 from 32. 4 from 62. 4 from 52. 4 from 42. 4 from 82. 4 from 72. 4 from 92.

10. 5 from 13. 5 from 23. 5 from 43. 5 from 53. 5 from 33. 5 from 63. 5 from 83. 5 from 73. 5 from 93.

11. 6 from 23. 6 from 43. 6 from 33. 6 from 53. 6 6 from 83. 6 from 93. 6 from 73.

12. 7 from 12. 7 from 22. 7 from 52. 7 from 72. 7 from 62. 7 from 42. 7 from 82. 7 from 92.

13. 8 from 94. 8 from 84. 8 from 74. 8 from 64. 8 from 54. 8 from 44. 8 from 34.

14. 9 from 85. 9 from 75. 9 from 65. 9 from 55. 9 from 45. 9 from 35. 9 from 25.

15. Count backward by *nines* from 90 to 0, as above.

16. Count backward by *tens* from 100 to 0.

SLATE EXERCISES.

Copy and subtract the following as above:

	(1.)	(2.)	(3.)	(4.)
From	736 pounds	674 yards	8567 hats	9678 dols.
Take	513 pounds	411 yards	4251 hats	8567 dols.

	(7.)	(8.)	(9.)	(10.)	(11.)
From	5876 in.	6341 oz.	7043 weeks	8672	9000
Take	2314 in.	1240 oz.	4043 weeks	5461	5000

MENTAL EXERCISES.

1. The age of a father is 50 years, and that of his son 20 years: how much older is the father than the son?

Analysis.—50 is 5 tens, and 20 is 2 tens; now 2 tens from 5 tens leave 3 tens, or 30. Therefore, etc.

2. 30 from 40 leaves how many?
3. 40 from 70 leaves how many? 20 from 40?
4. 24 from 47 leaves how many? 23 from 75?
5. 24 from 68 leaves how many? 35 from 47?
6. 32 from 65 leaves how many? 45 from 76?

SLATE EXERCISES.

When a figure in the Lower Number is *Larger* than the one above it.

1. Find the difference between 723 dols. and 476 dols.?

	Operation.
	723 dols.
	476 dols.
Ans.	247 dols.

1st Method.—Set down the numbers and begin at the right hand as before. Since 6 units cannot be taken from 3 units, we borrow 1 of the 2 tens and add it to the 3, making 13 units. Now 6 from 13 leaves 7, which we set under the figure subtracted. As we borrowed 1 of the 2 tens there is but 1 left; and 7 tens cannot be taken from 1 ten. We therefore borrow 1 of the 7 hundred and add it to the 1 ten, making 11 tens; and 7 from 11 leaves 4, which we set in tens' place. As we borrowed 1 of the 7 hundred, there are but 6 hundred left; and 4 from 6 leaves 2, which we set in hundreds' place.

Borrowing Illustrated by Unit Marks.

<table>
<tr><th></th><th>Hundreds.</th><th>Tens.</th><th>Units.</th></tr>
<tr><td>Tens borrowed,</td><td></td><td>10 tens.</td><td>10 units.</td></tr>
<tr><td>Minuend, 723=</td><td>| | | | | | |</td><td>| |</td><td>| | |</td></tr>
<tr><td>Subtrahend, 476=</td><td>| | | |</td><td>| | | | | | |</td><td>| | | | | |</td></tr>
<tr><td>Remainder, 247=</td><td>| |</td><td>| | | |</td><td>| | | | | | |</td></tr>
</table>

Analysis.—Let the 7 hundreds of the minuend be represented by 7 marks, the 2 tens by 2 marks, and the 3 units by 3 marks Let the subtrahend be represented in like manner.

Since we cannot take 6 units from 3 units, we borrow one of the 2 tens, which reduced to units, we add to the 3 units, making 13 units; and 6 from 13 leaves 7. Next, 7 tens cannot be taken from 1 ten (1 ten being erased and transferred to the units), we therefore borrow one of the hundreds, and add it to the 1 ten, making 11 tens; then 7 from 11 leaves 4. Finally, 4 hundreds from 6 hundreds (1 hundred being erased and transferred to the tens), leave 2 hundred. The result is 247 dollars.

2D METHOD.—As 6 units cannot be taken from 3 units, we add 10 to the 3, making 13; and 6 from 13 leaves 7, which we set under the figure subtracted. To balance the 10 added to the 3, instead of considering the next upper figure 1 less than it is, we add 1 ten to the 7 tens, the next figure in the lower number, making 8 tens. But 8 tens cannot be taken from 2 tens; we again add 10 to the 2, making 12 tens, and 8 from 12 leaves 4, which we set in tens' place. Finally, to balance the 10 added to 2, we add 1 to the next figure in the lower number, making 5 hundred, and 5 from 7 leaves 2, which we set under the figure subtracted. The result is 247 dols., the same as before.

10. What is adding 10 to the upper figure called?

Borrowing ten.

11. Why does not borrowing 10 affect the difference between the two numbers?

The ***First Method*** simply transfers a *unit* from a *higher* to the *next lower order* of the minuend; therefore *its value* is not altered.

By the ***Second Method*** the two numbers are *equally increased*; and when two numbers are equally increased, their *difference* is not altered.

NOTE.—This method is the less liable to mistakes, and is more generally practiced by business men.

12. How proceed by the second method, when the figure in the lower number is *larger* than the one above it?

Add 10 to the upper figure, then subtract, and add 1 to the next figure in the lower number.

	(2.)	(3.)	(4.)	(5.)	(6.)
From	4363	5830	7406	8738	9847
Take	2172	3517	5183	7329	8043

15. The preceding principles may be summed up in the following

GENERAL RULE.

I. *Place the less number under the greater, units under units, tens under tens, etc.*

II. *Begin at the right, and subtract each figure in the lower number from the one above it, setting the remainder under the figure subtracted.*

III. *If a figure in the lower number is larger than the one above it, add 10 to the upper figure; then subtract, and add 1 to the next figure in the lower number.*

PROOF.—*Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is right.*

NOTE.—This proof depends upon the Axiom that the *whole is equal to the sum of all its parts.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
From	465	6253	7464	6290
Take	230	3145	4273	6146

	(5.)	(6.)	(7.)	(8.)
From	5434	8670	7202	6290
Take	4260	3452	4101	4062

9. From 6435 quarts, take 4268 quarts.
10. From 265045 barrels, take 120328 barrels.

11. A farmer having 2568 bushels of corn, sold 1830 bushels: how many bushels had he left?

12. A's income is 2345 dollars, B's 3068 dollars: what is the difference between their incomes?

13. A man paid 1730 dollars for his horses, and 2135 dollars for his carriage: what was the difference in their cost?

14. What is the difference between nineteen hundred and nine, and nine hundred and nineteen?

15. What is the difference between two thousand and four, and one thousand and fourteen?

16. Find the difference between eight hundred and eight, and eight thousand and eighty.

17. 7800461 — 4560231. 18. 8000030 — 6234521.

19. 7930451 — 4000459. 20. 9603245 — 2896750.

21. 6235672 — 4000563. 22. 1900000 — 899996.

23. If a farmer has 738 sheep, how many more must he buy to make up 1320?

24. A man bought goods for 1943 dollars, and sold the same for 2365 dollars: what did he gain:

25. A man born in 1783, died in 1866: how old was he?

26. A person bought a drove of cattle for 5263 dollars, and sold them for 4675 dollars: how much did he lose?

27. The difference between the ages of two persons is 15 years, and the older is 79 years: how old is the younger?

28. 1463 and what number make 3185?

29. A has 765 dollars, B 1695 dollars, and C's money was equal to the difference between A's and B's: how much money had C?

30. The Pilgrim Fathers landed at Plymouth Rock in 1620, and the independence of the colonies was declared in 1776: how many years between these two events?

DRILL FOR RAPID COMBINATIONS.

To TEACHERS.—These exercises, if properly conducted, will secure two objects: *First*, the *habit of fixing the attention*; *Second*, *rapidity* in the combination of numbers. They should be dictated slowly at first, increasing in speed as the class acquire ability to follow. The answers may be given individually, or by the class simultaneously.

ORAL.—1. From 12, subtract 5; add 6; subtract 4; add 3; add 4; subtract 10; add 9; subtract 4; add 2; subtract 5: what is the result?

EXPLANATION.—The teacher says, "from 12 subtract 5," the class think 7; "add 6," the class think 13; "subtract 4," the class think 9; "add 3," the class think 12, and so on.

2. To 9, add 4; subtract 2; add 5; subtract 6; add 3; subtract 5; add 7; add 3; subtract 5: the result?

3. From 17 take 15; add 10; take 2; add 9; add 6; take 5; take 10; add 7; take 2: result?

4. To 23 add 5; take 6; add 9; add 4; take 10; add 9; take 4; add 7; add 10; take 9; add 4: result?

5. From 24 take 6; add 8; take 10; add 5; add 7; take 3; add 7; take 6; add 5; add 3: result?

6. To 35 add 4; take 6; take 3; add 8; take 7; add 5; add 3; take 9; add 7; take 8; add 10: result?

SLATE.—1. To 375 add 123; subtract 47; add 23; add 47; subtract 36; add 87; subtract 68: the result?

2. From 62 take 34; add 76; take 40; add 78; take 99; add 76; add 24; take 43: result?

3. Add 344 to 65; take 64; add 784; take 678; add 407; take 309; add 860: result?

4. From 780 take 607; add 788; add 28; take 19; add 976; take 306; add 1000: result?

5. To 4678 add 6246; take 4004; add 5020; take 508; add 1700; take 468; add 2500: result?

6. From 8640 take 3476; add 4578; take 5065; add 87; take 1000; add 608; take 47: result?

MULTIPLICATION.

MENTAL EXERCISES.

To Teachers.—The object of this Exercise is to *develop the idea of "times,"* as used in *Multiplication*, preparatory to learning the Table.

1. If your father gives you 3 books at one time, and 3 at another, how many books will you have?

"3 books and 3 books are 6 books."

2. How many times 3 books will you have?

"Two times."

3. How many are 2 times 3 books? "6 books."

4. How many are 2 times 2 pencils?

5. Show this by counters or unit marks.

6. How many are 2 cents, and 2 cents, and 2 cents?

7. How many are 3 times 2 cents? Show it.

8. If you have 4 fingers on each hand, how many have you on both hands?

9. How many are 2 times 4? Show it.

10. John has 5 apples, and Henry has 2 times as many: how many has Henry? Show it.

11. If 1 orange costs 6 cents, what will 2 oranges cost?

Analysis.—If 1 orange costs 6 cents, 2 oranges will cost 2 times 6 cents; and 2 times 6 cents are 12 cents. Therefore, 2 oranges will cost 12 cents.

12. How many are 7 slates and 7 slates? Show it.

13. If 1 yard of braid costs 7 cents, what will 2 yards cost? Show it.

14. At 8 cents each, what will be the cost of 2 inkstands? Show it.

15. If 1 writing-book costs 9 cents, what will 2 writing-books cost? Show it.

16. How many are 2 times 10 dollars? Show it.

MULTIPLICATION TABLE.

once			2 times			3 times			4 times		
1	is	1	1	are	2	1	are	3	1	are	4
2	"	2	2	"	4	2	"	6	2	"	8
3	"	3	3	"	6	3	"	9	3	"	12
4	"	4	4	"	8	4	"	12	4	"	16
5	"	5	5	"	10	5	"	15	5	"	20
6	"	6	6	"	12	6	"	18	6	"	24
7	"	7	7	"	14	7	"	21	7	"	28
8	"	8	8	"	16	8	"	24	8	"	32
9	"	9	9	"	18	9	"	27	9	"	36
10	"	10	10	"	20	10	"	30	10	"	40
11	"	11	11	"	22	11	"	33	11	"	44
12	"	12	12	"	24	12	"	36	12	"	48

5 times			6 times			7 times			8 times		
1	are	5	1	are	6	1	are	7	1	are	8
2	"	10	2	"	12	2	"	14	2	"	16
3	"	15	3	"	18	3	"	21	3	"	24
4	"	20	4	"	24	4	"	28	4	"	32
5	"	25	5	"	30	5	"	35	5	"	40
6	"	30	6	"	36	6	"	42	6	"	48
7	"	35	7	"	42	7	"	49	7	"	56
8	"	40	8	"	48	8	"	56	8	"	64
9	"	45	9	"	54	9	"	63	9	"	72
10	"	50	10	"	60	10	"	70	10	"	80
11	"	55	11	"	66	11	"	77	11	"	88
12	"	60	12	"	72	12	"	84	12	"	96

9 times			10 times			11 times			12 times		
1	are	9	1	are	10	1	are	11	1	are	12
2	"	18	2	"	20	2	"	22	2	"	24
3	"	27	3	"	30	3	"	33	3	"	36
4	"	36	4	"	40	4	"	44	4	"	48
5	"	45	5	"	50	5	"	55	5	"	60
6	"	54	6	"	60	6	"	66	6	"	72
7	"	63	7	"	70	7	"	77	7	"	84
8	"	72	8	"	80	8	"	88	8	"	96
9	"	81	9	"	90	9	"	99	9	"	108
10	"	90	10	"	100	10	"	110	10	"	120
11	"	99	11	"	110	11	"	121	11	"	132
12	"	108	12	"	120	12	"	132	12	"	144

DEFINITIONS.

1. What is Multiplication?

Multiplication is finding the *amount* of a number taken or added to itself a given number of times.

2. What is the number to be multiplied called?

The ***Multiplicand.***

3. What the number by which you multiply?

The ***Multiplier;*** and shows how many times the multiplicand is to be taken.

4. What is the number obtained by multiplication called?

The ***Product.***

When it is said that 3 times 4 are 12, which is the multiplicand? The multiplier? The product?

When it is said that 4 times 3 are 12, what is the 4? The 3? The 12?

5. What else are the multiplier and multiplicand called?

Factors; for they *make* or *produce* the product. The number 12 is made up of four 3s, or three 4s; hence, 3 and 4 are factors of 12.

REMARK.—The *product* is the *same* in whatsoever order the factors are multiplied. Thus, if 4 be represented by a horizontal row of unit marks upon the blackboard, and 3 by a perpendicular row of 3 unit marks, it is plain that the horizontal row taken 3 times, is equal to the perpendicular row taken 4 times.

✱ ✱ ✱ ✱
✱ ✱ ✱ ✱
✱ ✱ ✱ ✱

6. When the multiplicand contains only one denomination, what is the operation called?

Simple Multiplication.

7. How is Multiplication denoted?

By an ***oblique cross,*** called the *sign of multiplication* ($\times$). Thus, 6×4 shows that 6 and 4 are to be multiplied together, and is read "6 times 4," "6 into 4," or "6 multiplied by 4."

Read the following expressions: $2 \times 6 = 3 \times 4$; $4 \times 5 = 10 \times 2$; $2 \times 2 \times 6 = 12 \times 2$.

MENTAL EXERCISES.

1. What will 4 pears cost, at 3 cents apiece?

ANALYSIS.—Since 1 pear costs 3 cents, 4 pears will cost 4 times 3 cents; and 4 times 3 cents are 12 cents. Therefore, 4 pears will cost 12 cents.

*** It is important for the pupil to analyze every concrete example in a concise, distinct, and scholarly manner.

2. What will 5 oranges cost, at 6 cents apiece?
3. If 1 hat costs 6 dollars, what will 4 hats cost?
4. At 7 dollars a barrel, how much will 3 barrels of flour come to?
5. If you obtain 6 credit-marks each day for 5 days in succession, how many will you have?
6. In 1 week there are seven days: how many days are there in 6 weeks?
7. George has 7 marbles, and Henry has 4 times as many: how many marbles has Henry?
8. At 8 cents apiece, what will 6 tops cost?
9. At 9 dollars each, what will 4 trunks cost?

SLATE EXERCISES.

Copy and multiply the following, setting each result under the figure multiplied:

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
Mult.	6	7	8	9	7	5	8	9
By	5	4	6	5	6	7	6	7
Prod.	30			45				63

	(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)
Mult.	8	9	7	9	8	9	10	11
By	7	8	9	6	9	7	8	9

MENTAL EXERCISES.

1. Bought 7 barrels of flour, at 6 dollars a barrel: what did the flour come to?

2. Sold 8 silk umbrellas, at 7 dollars each: what was the amount of the bill?

3. If a man earns 9 dollars a week, how much will he earn in 8 weeks?

4. What must be paid for 6 quarts of cherries, at 12 cents a quart?

5. In a certain orchard there are 8 rows of trees, and 12 trees in a row: how many trees does it contain?

6. If 1 table costs 8 dollars, what will be the cost of 10 tables?

7. What must I pay for 11 yards of muslin, which is 12 cents a yard?

8. How many quarts in 11 pecks, allowing 8 quarts to a peck?

9. In 1 dime there are 10 cents: how many cents are there in 11 dimes?

10. In 1 year there are 12 months: how many months are there in 10 years?

SLATE EXERCISES.

When the Multiplier has but *one* figure, and the Product of each figure in the Multiplicand is *Less* than 10.

1. Multiply 1232 by 3.

OPERATION.

$$\begin{array}{r} 1232 \\ 3 \\ \hline 3696 \end{array}\ \textit{Ans.}$$

ANALYSIS.—Write the multiplier under the multiplicand, and begin at the right. 3 times 2 units are 6 units. Set the 6 in units place, under the figure multiplied, because it is *units*. 3 times 3 tens are 9 tens. Set the 9 in tens place, because it is *tens*. 3 times 2 hundreds are 6 hundreds. Set the 6 in hundreds place, because, etc. 3 times 1 thousand are 3 thousands. Set the 3 in thousands place.

Copy and multiply the following in like manner:

	(2.)	(3.)	(4.)	(5.)
Mult.	42414	22321	12212	11111
By	2	3	4	5

	(6.)	(7.)	(8.)	(9.)
Mult.	101101	332032	110101	111111
By	6	3	7	8

MENTAL EXERCISES.

1. At 9 dollars a barrel, what will 8 barrels of cranberries come to?

2. What will be the cost of 6 flutes, at 12 dollars each?

3. A farmer sold 11 calves, at 9 dollars apiece: what did he receive for them?

4. How many are 9 times 7? 8 times 9?

5. Bought 10 accordions, at 12 dollars each: what was the amount of the bill?

6. How many are 8 times 7? 9 times 8?

7. If 1 plough cost 11 dollars, what will be the cost of 12 ploughs, at the same rate?

8. How many are 11 times 10? 12 times 11?

WRITTEN EXERCISES.

When the Product of the respective figures is 10, or *More*.

1. If 1 horse costs 435 dollars, how much will 3 horses cost?

OPERATION.	
435	mult'd.
3	mult.
1305	dols.

ANALYSIS.—Since 1 horse costs 435 dollars, 3 horses will cost 3 times as much. Write the multiplier under the multiplicand, and beginning at the right, proceed thus: 3 times 5 units are 15 units; we set the 5 in units place, under the figure by which we multiply, and carry the 1 to the product of the next figure, as in addition. Next, 3 times 3 tens

are 9 tens, and 1 (to carry) makes 10 tens; we set the 0 in tens place, and carry the 1 to the product of the next figure. Finally, 3 times 4 hundred are 12 hundred, and 1 (to carry) makes 13 hundred. Therefore, 3 horses will cost 1305 dollars.

7. How write numbers for multiplication?

Write the multiplier under the multiplicand, units under units, etc.

8. How proceed when the multiplier contains but *one* figure?

Begin at the right hand, and multiply each figure in the multiplicand by the multiplier, separately.

9. What do you do with the partial results, when 10, or more?

Set the *units figure* under the figure multiplied, and carry the *tens* to the product of the next figure.

10. When the multiplier is units, what order is the product?

The same order as the figure multiplied.

11. What are the principles as to the nature of the multiplier, the multiplicand, and the product?

1st. The ***Multiplier*** must always be considered an *abstract* number.

2d. The ***Multiplicand*** may be an *abstract*, or *concrete* number.

3d. The ***Product*** is always the *same name* or *kind* as the true multiplicand; for, repeating a number does not change its nature.

12. Which of the factors is the true multiplicand?

The ***true multiplicand*** is that number, which added to itself the given number of times, will produce the required product.

REMARK.—Neither a *concrete* nor *abstract* number can properly be said to be repeated as many times as another is *long*, or *heavy*. Hence, money can not be multiplied by yards, pounds, etc.; but any given sum can be multiplied by a *number of units* equal to the number of yards, pounds, etc., in the given quantity, and the product will be money.

2. In 1 year there are 365 days: how many days are there in 5 years?

3. If 1 piano costs 750 dollars, what will 6 pianos cost?

4. If 1 farm contains 875 acres, how many acres will 8 farms of the same size contain?

	(5.)	(6.)	(7.)	(8.)
Mult.	2136	7345	28536	65043
By	2	3	4	5

	(9.)	(10.)	(11.)	(12.)
Mult.	701230	635728	830405	973080
By	6	7	8	9

MENTAL EXERCISES.

1. What will 3 tables cost, at 45 dollars apiece?

ANALYSIS.—3 tables will cost 3 times as much as 1 table. But 45 is equal to 4 tens and 5 units. Now 3 times 4 tens are 12 tens, or 120; 3 times 5 units are 15 units, or 1 ten and 5 units; and 1 ten and 5 units added to 120 make 135. Therefore, 3 tables will cost 135 dollars.

NOTE.—When the numbers to be multiplied *mentally* are large, it is advisable to separate them into the units, tens, etc., of which they are composed, and multiply the highest order first, then the next lower, etc., adding the results as we proceed. (P. 26, N.)

2. How much can a man earn in 2 months, if he earns 36 dollars a month?

3. In a peach orchard there are 5 rows of trees, and 27 trees in a row: how many peach trees does the orchard contain?

4. How many are 3 times 54? 4 times 37?

5. If a man can earn 56 dollars a month, how much can he earn in 7 months?

6. If 1 hogshead contains 63 gallons of molasses, how many gallons will 5 hogsheads contain?

7. If 1 melodeon can be had for 75 dollars, what will be the cost of 6 melodeons?

8. If 1 sofa is worth 83 dollars, how much are 9 sofas worth?

WRITTEN EXERCISES.

When the Multiplier has *two* or *more* Figures.

1. What will 106 buggies cost, at 268 dollars apiece?

OPERATION.

```
     268 mult'd.
     106 mult.
    ----
    1608
   268
   -----
Ans. 28408 dols.
```

ANALYSIS.—106 buggies will cost 106 times as much as 1 buggy. We write the multiplier under the multiplicand, as before, and beginning at the right hand, proceed thus: 6 times 8 units are 48 units. The 8 is set in *units* place, under the figure which produced it, because it is *units*; and the 4 is carried to the product of the next figure, because it is the *same order* as that figure. The other figures of the multiplicand are multiplied by 6, and the results set down in a similar manner. Next, the product by 0 tens is 0; we therefore omit it. Again, 1 hundred times 8 units are 8 hundreds. The 8 is set in hundreds place, under the figure which produced it, *because it is hundreds.* The other figures of the multiplicand are multiplied by 1 in the same manner. Finally, adding these partial products together, the result, 28408 dollars, is the *whole* product required.

13. When the multiplier has two or more figures, how proceed?

Beginning at the *right hand,* multiply the multiplicand by each figure of the multiplier *separately,* and set the *first figure* of each *partial product* under the multiplying figure.

14. What is meant by partial products?

They are the *several results* which arise from multiplying the multiplicand by the separate figures of the multiplier, and are so called because they are *parts* of the whole product.

15. What is done with the partial products, and why?

We add them together, because the *whole product* is equal to the *sum* of *all its parts.*

	(2.)	(3.)	(4.)	(5.)	(6.)
Mult.	3724	4103	5378	6037	8734
By	25	34	46	57	78

16. The preceding principles may be summed up in the following

GENERAL RULE.

I. *Place the multiplier under the multiplicand, units under units, tens under tens,* etc.

II. *When the multiplier has but one figure, beginning at the right, multiply each figure of the multiplicand by it, and set down the result as in addition.*

III. *If the multiplier has two or more figures, multiply the multiplicand by each figure of the multiplier separately, and set the first figure of each partial product under the multiplying figure.*

Finally, the sum of the partial products will be the answer required.

PROOF.—*Multiply the multiplier by the multiplicand; if this result agrees with the first, the work is right.*

NOTE.—This proof is based upon the principle, that the result will be the same whichever of the given numbers is taken as the multiplicand. (P. 47, Q. 5.)

EXAMPLES FOR PRACTICE.

1. Multiply 78 by 43, and prove the operation?

OPERATION.		PROOF.	
Multiplicand	78	The given multiplier	43
Multiplier	43	" " multiplicand	78
	234		344
	312		301
Product	3354	The same as the first	3354

(2.)	(3.)	(4.)	(5.)
27356	40256	57189	70203
21	27	32	47

(6.)	(7.)	(8.)	(9.)
631420	507060	813670	973848
158	249	365	1476

10. There are 24 hours in a day: how many hours are there in 365 days?

11. There are 320 rods in a mile: how many rods are in 150 miles?

12. What will 265 acres of land cost at 87 dollars per acre?

13. What cost 97 melodeons, at 250 dollars apiece?

14. Multiply 43846 by 123.

15. Multiply 57028 by 321.

16. Multiply 604326 by 237.

17. Multiply 673862 by 250.

18. Multiply 703562 by 304.

19. Multiply 570031 by 402.

20. Multiply 439275 by 425.

21. Multiply 789426 by 521.

22. What will be the cost of 85 pianos, at 650 dollars apiece?

23. If a ship sails 115 miles in one day, how far will she sail in 198 days?

24. If there are 63 yards in 1 piece of cloth, how many are there in 268 pieces?

25. At 320 dollars a yoke, what will 500 yoke of oxen cost?

26. What will 110 wagons cost, at 175 dollars apiece?

27. What cost 350 suits of clothes, at 115 dollars a suit?

28. What cost 1645 saddles, at 75 dollars apiece?

29. What cost 3250 tons of iron, at 87 dollars a ton?

30. What does the President's salary amount to in 8 years, at 25000 dollars a year?

31. What is the expense of furnishing an army of 11500 men with uniforms which cost 57 dollars apiece?

32. If 1 ox weighs 1163 pounds will 100 oxen weigh?

33. What cost 465 velvet cloaks, at 129 dollars apiece?

34. What cost 1567 tons of lead, at 120 dollars per ton?

CONTRACTIONS.

I. When the Multiplier is 10, 100, 1000, etc.

17. What is the effect of annexing a cipher to a number?

Annexing *one cipher* to a number multiplies it by 10; annexing *two ciphers* multiplies it by 100, and so on.

REMARK.—The learner will observe that each cipher annexed to a number, removes each preceding figure in the number to the *next higher order*, which has *ten* times the value of the order from which it has been removed. (Page 11, Q. 17.)

2. What will 10 sofas cost, at 56 dollars apiece?

SOLUTION.—Annexing a cipher to 56 dollars, the result is 560 dollars, which is the cost required.

3. What will 100 acres of land cost, at 75 dollars an acre?

SOLUTION.—Annexing 2 ciphers to 75 dollars, the result is 7500 dollars, which is the answer required.

18. How then do you multiply by 10, 100, 1000, etc.?

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the result will be the product.

4. What is the product of 361 multiplied by 100?
5. Multiply 453 by 100.
6. Multiply 2045 by 1000.
7. Multiply 46208 by 1000.
8. Multiply 58241 by 1000.
9. Multiply 326072 by 10000.
10. Multiply 4007289 by 100000.
11. What cost 10 cows, at 51 dollars apiece?
12. At 265 dollars apiece, what will 100 buggies come to?
13. What cost 100 acres of land, at 205 dollars per acre?
14. If 1 bushel of apples is worth 63 cents, what will be the price of 1000 bushels?

II. When one or both Factors have Ciphers on the right.

15. If 1 railroad car costs 2700 dollars, what will 50 cars cost?

OPERATION.

$$\begin{array}{r} 2700 \\ 50 \\ \hline \end{array}$$

Ans. 135000 dols.

ANALYSIS.—If 1 car cost 2700 dollars, 50 cars will cost 50 times as much. We resolve the multiplicand into the factors 27 and 100; and the multiplier into 5 and 10. Now, as the product is the same in whatever order the factors are taken, omitting the ciphers on the right of the multiplicand and multiplier, we multiply the significant figures together as before, and annex the ciphers omitted to the product. The result is 135000 d.

19. How proceed when one or both factors have ciphers on the right?

Multiply the significant figures together; and to the result annex as many ciphers as are found on the right of both factors.

	(16.)	(17.)	(18.)
Mult.	1860	25000	4053
By	300	7	2000
Prod.	558000	175000	8106000

19. Multiply 37000 by 31.
20. Multiply 52300 by 65.
21. Multiply 42721 by 2000.
22. Multiply 60045 by 3100.
23. Multiply 85000 by 2300.
24. Multiply 375000 by 57000.
25. Multiply 204200 by 20500.
26. Multiply 800400 by 600300.
27. What will 200 acres of land cost, at 70 dollars per acre?
28. What cost 21000 bushels of oats, at 60 cents a bushel?
29. If a man travels 120 miles a day, how far can he travel in 300 days?
30. If 1 acre produces 50 bushels of corn, what will 3000 acres produce?
31. Multiply 25 thousand by 25 hundred.
32. Multiply two hundred and forty-five thousand by 16 thousand.
33. Multiply 65 thousand and seventy by 21 thousand seven hundred.
34. Multiply one million, one hundred and ten thousand, by 26 thousand.

QUESTIONS FOR REVIEW.

ORAL.—1. If 9 men can build a wall in 12 days, how long will it take 1 man to build it?

ANALYSIS.—It will take 1 man 9 times as long as 9 men, and 9 times 12 days are 108 days. Therefore, it will take 1 man 108 days.

2. If a jar of butter will last a family of 8 persons 6 weeks, how long will it last 1 person?

3. Henry can read a book through in 11 days by reading 6 hours each day: how long will it take him if he reads 1 hour a day?

4. If 12 men can frame a house in 8 days, how long will it take 1 man to frame it?

5. If I buy 4 barrels of apples at 3 dollars a barrel, and 4 barrels of pears at 5 dollars, what will be the cost of both?

6. A farmer having 15 bushels of wheat, sold 9 bushels at 2 dollars a bushel, and the remainder at 3 dollars a bushel: how much did he get for his wheat?

WRITTEN.—1. If it takes 285 laborers 18 months to build a railroad, how long would it take 1 man to build it?

2. A ship of war has provisions to last a crew of 625 men 90 days: how long would they last 1 man?

3. If a clerk has 36 dollars a month for the first 4 months; 48 dollars a month for the next 4; and 60 dollars a month for the next 4; what will he receive for the year?

4. A man having 1000 dollars in his pocket, gave 45 dollars each to 12 poor persons: how much had he left?

5. If I receive 150 dollars a month, how much shall I have at the end of the year, after deducting 28 dollars a month for board?

DIVISION.

MENTAL EXERCISES.

10 TEACHERS.—The object of this preliminary Exercise is to *develop the idea of "times,"* as used in *Division*, preparatory to learning the Table.

1. If I have 9 pencils, how many boys can I supply with 3 pencils each?

ANALYSIS.—If I give one boy 3 pencils, how many will be left? "Six pencils."

If I give another boy 3, how many pencils will be left? "Three."

If I give another 3, how many will be left? "None."

How many boys have I supplied with 3 pencils? "Three."

How many times are 3 pencils contained in 9 pencils?

"Three times."

2. How many peaches, at 2 cents each, can you buy for 8 cents? Show this by counters.

3. How many oranges, at 4 cents each, can you buy for 12 cents? How many times 4 make 12? Show this.

4. In 1 gallon there are 4 quarts: how many gallons are there in 8 quarts? How many times 4 make 8? Show this by unit marks.

5. If a lad earns 5 dollars a week, how long will it take him to earn 25 dollars? How many times 5 make 25? Show this.

6. At 6 cents an ounce, how many ounces of candy can you buy for 18 cents? Show this by unit marks.

7. How many lambs, at 2 dollars apiece, can be had for 20 dollars? Show this.

8. At 4 dollars a pair, how many pair of boots can I buy for 16 dollars? Show this.

9. If I have 20 pounds of flour, how many poor persons can I supply with 5 pounds each?

DIVISION TABLE.

1 is in		2 is in		3 is in		4 is in	
1,	once.*	2,	once.	3,	once.	4,	once.
2,	2	4,	2	6,	2	8,	2
3,	3	6,	3	9,	3	12,	3
4,	4	8,	4	12,	4	16,	4
5,	5	10,	5	15,	5	20,	5
6,	6	12,	6	18,	6	24,	6
7,	7	14,	7	21,	7	28,	7
8,	8	16,	8	24,	8	32,	8
9,	9	18,	9	27,	9	36,	9
10,	10	20,	10	30,	10	40,	10

5 is in		6 is in		7 is in		8 is in	
5,	once.	6,	once.	7,	once.	8,	once.
10,	2	12,	2	14,	2	16,	2
15,	3	18,	3	21,	3	24,	3
20,	4	24,	4	28,	4	32,	4
25,	5	30,	5	35,	5	40,	5
30,	6	36,	6	42,	6	48,	6
35,	7	42,	7	49,	7	56,	7
40,	8	48,	8	56,	8	64,	8
45,	9	54,	9	63,	9	72,	9
50,	10	60,	10	70,	10	80,	10

9 is in		10 is in		11 is in		12 is in	
9,	once.	10,	once.	11,	once.	12,	once.
18,	2	20,	2	22,	2	24,	2
27,	3	30,	3	33,	3	36,	3
36,	4	40,	4	44,	4	48,	4
45,	5	50,	5	55,	5	60,	5
54,	6	60,	6	66,	6	72,	6
63,	7	70,	7	77,	7	84,	7
72,	8	80,	8	88,	8	96,	8
81,	9	90,	9	99,	9	108,	9
90,	10	100,	10	110,	10	120,	10

* After 2, 3, 4, etc., in the second column, "times" is understood.

DEFINITIONS.

1. What is Division?

Division is finding how many times one number is contained in another.

2. What is the number to be divided called?

The ***Dividend.***

3. The number to divide by?

The ***Divisor.***

4. What is the number obtained by division called?

The ***Quotient.***

5. What is the number *left* called?

The ***Remainder.***

When it is said that 3 is contained in 13, 4 times and 1 over, which is the dividend? The divisor? The quotient? The remainder?

REMARKS.—1. The remainder is always the *same denomination* as the *dividend;* for, it is *a part* of the dividend not yet divided.

2. A *proper* remainder is always *less* than the divisor.

6. When the dividend contains *only one denomination,* what is the operation called?

Simple Division.

7. How is Division denoted?

By a ***short horizontal line*** between two dots (÷), called the *Sign of division.*

8. When placed between two numbers what does it show?

It shows that the number *before* it is to be divided by the one *after* it. Thus, $21 \div 3$, shows that 21 is to be divided by 3, and is read "21 divided by 3."

9. How else is division denoted?

By *writing the divisor under the dividend* with a short line between them; as $\frac{21}{3}$.

Read the following: $9 \div 3 = 3$; $24 \div 4 = 5 + 1$; $39 \div 3 = 10 + 3$; $5 + 4 = 36 \div 4$; $\frac{28}{4} = 7$; $\frac{35}{5} = 4 + 3$.

OBJECTS OF DIVISION.

1. A lad having 6 cents wishes to buy pears, which are 2 cents apiece: how many can he buy?

ILLUSTRATION.

✱ ✱ | ✱ ✱ | ✱ ✱

ANALYSIS.—He can buy as many pears as there are times 2 cents in 6 cents. The object then is to find *how many times* 2 is contained in 6; and 2 is in 6, 3 times.

2. A lad has 6 pears, which he wishes to divide equally between 2 companions: how many can he give to each?

ILLUSTRATION.

✱ ✱ ✱ | ✱ ✱ ✱

ANALYSIS.—The object of this example is to *divide* 6 pears into 2 *equal parts*. Dividing 6 by 2, the quotient is 3, which shows that there are 3 pears in each part.

10. What is the object or office of Division?

Its ***object or office*** is *twofold: First,* To find *how many times* one number is contained in another. (Ex. 1.)

Second, To *divide* a number into *equal parts.* (Ex. 2.)

REMARK.—The two preceding examples are *representatives* of the two classes of problems to which Division is applied. In the *first* class, the divisor and dividend are always of the *same denomination,* and the quotient is *times,* or an *abstract* number.

In the *second,* the divisor and dividend are of *different denominations,* and the *quotient* is always of the same denomination as the *dividend.* This class involves the *idea* of Fractions, and will receive further attention under that branch of the science.

NOTE.—The process of reasoning in the solution of these two classes of examples is somewhat different; but the practical operation is the same, viz.: to *find how many times* one number is contained in another, which accords with the definition of Division.

11. How divide a number into *two, three, four,* etc., equal parts?

Divide the number by 2, 3, 4, 5, etc., respectively.

12. When a thing is divided into 2, 3, 4, etc., equal parts, what are the parts called?

If divided into *two equal parts,* the parts are called *halves;* into *three,* the parts are called *thirds;* into *four,* they are called *fourths;* into *five, fifths;* etc.

13. When a thing is divided into *equal parts*, from what do the parts take their name?

From the *number of parts* into which the thing is divided.

3. What is a half of 10? A third of 12? A fourth of 16? A fifth of 20? A sixth of 30?

4. What is a seventh of 35? An eighth of 56? A ninth of 45? A tenth of 60? A twelfth of 84?

SHORT DIVISION.

MENTAL EXERCISES.

1. How many lemons, at 2 cents apiece, can George buy for 10 cents?

ANALYSIS.—Since 2 cents will buy 1 lemon, 10 cents will buy as many lemons as 2 is contained times in 10; and 2 is in 10, 5 times. Therefore, he can buy 5 lemons.

2. At 4 cents each, how many bananas can you buy for 12 cents?

3. How many yards of tape, at 6 cents a yard, can be had for 18 cents?

4. At 4 dollars a yard, how many yards of cloth can you buy for 28 dollars?

5. When the fare on the city railroads is 5 cents a ride, how many rides can you take for 30 cents?

6. If 3 oranges cost 12 cents, what will 1 cost?

ANALYSIS.—If 3 oranges cost 12 cts., 1 orange will cost 1 third of 12 cts.; and 1 third of 12 cts. is 4 cts. (P. 63, Q. 12.)

7. If 5 slates cost 60 cents, what will 1 cost?

8. A baker divided 28 loaves of bread equally among 7 beggars: how many loaves did he give to each?

9. A grocer sold 9 barrels of flour for 72 dollars: what was that a barrel?

SLATE EXERCISES.

When the Divisor is exactly contained in each figure of the Dividend.

1. How many times is 2 contained in 6402?

Analysis.—Write the divisor on the left of the dividend, with a curve line between them, and proceed thus: 2 is contained in 6, 3 times; write the 3 under the figure divided, for it is the *same order* as that figure. Next, 2 is contained in 4, 2 times; write the 2 under the figure divided, for the same reason as before. 2 is contained in 0, no times; write a cipher in the quotient. Finally, 2 is in 2, 1 time; set the 1 under the figure divided.

Operation.

$$2\overline{)6402}$$

Ans. 3201

10. How write numbers for division?

Place the divisor on *the left* of the dividend, with a *curve line* between them.

11. How proceed when the divisor is contained exactly in each figure of the dividend?

Begin at the left of the dividend, and divide each figure by the divisor; placing the result under the figure divided.

12. What order is each quotient figure?

The *same order* as the figure divided.

Copy and divide the following in like manner:

(2.)	(3.)	(4.)	(5.)
3)6393	2)4062	4)8404	5)50505
(6.)	(7.)	(8.)	(9.)
4)8084	6)6606	7)7070	8)80808

MENTAL EXERCISES.

1. At 4 dollars a head, how many sheep can a man buy with 35 dollars, and what will he have left?

Analysis.—4 dollars are contained in 35 dollars 8 times, and 3 over. Therefore, he can buy 8 sheep, and have 3 dollars left.

2. How many times is 3 contained in 17, and how many over?

3. In 24 how many times 5, and how many over?

4. In 39 how many times 4? 5? 6? 7? 8? 9?

5. How many boxes, each containing 6 quarts, can be filled with 40 quarts of blue-berries?

6. Horace has 38 marbles, which he wishes to distribute equally among his 3 brothers: how many can he give to each; and how many over?

7. How many times 7 in 29, and how many over?

8. How many times 8 in 57? In 63? In 74? In 83?

SLATE EXERCISES.

When the Divisor is not contained exactly in each figure of the Dividend.

1. How many barrels of flour, at 5 dollars a barrel, can be bought for 157034 dollars?

OPERATION.

$$5)\underline{157034}$$

Ans. $314068\frac{4}{5}$

ANALYSIS.—As the divisor is not contained in the first figure of the dividend, we must find how many times it is contained in the first two figures, which is 3 times, and set the 3 under the right hand figure divided. Again, 5 is contained in 7, once and 2 remainder. Set the 1 under the figure divided, and prefixing the 2 remainder mentally to the next figure of the dividend makes 20. Now 5 is in 20, 4 times. Set the 4 under the figure divided. Next, 5 is not contained in 3, the next figure of the dividend; we therefore put a *cipher* in the quotient, and prefixing the 3 mentally to the next figure of the dividend, makes 34. Now, 5 is in 34, 6 times and 4 remainder. We set the 6 under the figure divided, and as there are no more figures in the dividend, we write this last remainder over the divisor, and annex it to the quotient.

13. When the divisor is not contained in the first figure of the dividend, how proceed?

Find how many times it is contained in the *first two figures.*

14. When it is not contained in a subsequent figure of the dividend, how?

Put a *cipher* in the quotient, and find how many times the divisor is contained in this and the next *figure*, setting the result under the *right hand figure divided.*

15. When there is a remainder after dividing a figure, how?

Prefix it mentally to the *next figure* of the dividend, and divide this number as before.

16. If there is a remainder after dividing the last figure of the dividend, what is to be done?

Write it *over* the divisor, and *annex* it to the quotient?

NOTE.—To *prefix* signifies to place *before;* to *annex,* to place after.

17. What are the principles as to the nature of the divisor and dividend, the quotient and remainder?

1st. The **divisor and dividend** may be *abstract* or *concrete* numbers.

2d. When they are the *same denomination,* the quotient denotes *times,* and is an *abstract* number.

3d. When they are *different* denominations, the quotient denotes *equal parts,* and is the *same denomination* as the dividend.

4th. The *remainder* is always the *same denomination* as the dividend; for, it is an *undivided* part of it.

18. What is Short Division?

Short Division is the method of dividing, when the *results* of the several steps are carried in the mind, and the *quotient* only is set down.

Copy and divide the following by Short Division:

(2)	(3.)	(4.)	(5.)
4)12568	3)60429	6)18728	7)84079

19. The preceding principles may be summed up in the following

RULE FOR SHORT DIVISION.

I. *Place the divisor on the left of the dividend, and beginning at the left, divide each figure by it, setting the result under the figure divided.*

II. *If the divisor is not contained in a figure of the dividend, put a cipher in the quotient, and find how many times the divisor is contained in this and the next figure, setting the result under the right hand figure divided.*

III. *If a remainder arises from any figure before the last, prefix it mentally to the next figure, and divide as before.*

If from the last, place it over the divisor, and annex it to the quotient.

PROOF.—*Multiply the divisor and quotient together, and to the product add the remainder. If the result is equal to the dividend, the work is right.*

NOTE.—This proof depends upon the principle, that *Division* is *the reverse of Multiplication;* the *dividend* answering to the *product*, the *divisor* to *one* of the factors, and the *quotient* to the *other*.

EXAMPLES FOR PRACTICE.

1. At 8 dollars apiece, how many hats can be bought for 23243 dollars?

OPERATION.	PROOF.
8)23243	$2905 \times 8 = 23240$
Quot. 2905, 3 over.	Add the remainder, 3
Ans. 2905 hats, and 3 dols over.	Dividend 23243

(2.)	(3.)	(4.)	(5.)
2)23416	3)34169	4)48016	5)90310

(6.)	(7.)	(8.)	(9.)
6)67419	7)75008	8)89619	9)93048

10. How many barrels of apples, at 3 dollars a barrel, can you buy for 846 dollars?

11. At 5 dollars apiece, how many hats can be bought for 2300 dollars?

12. At 6 dollars a barrel, how many barrels of flour will 3522 dollars buy?

13. At 7 days each, how many weeks in 365 days?

14. If a man travels 8 miles per hour, how long will it take him to travel 1000 miles?

15. If 1 boat will carry 9 persons over a river, how many boats will it require to carry 468 persons over?

16. If a man lays up 12 dollars a week, how long will it take him to lay up 288 dollars?

17. At 11 dollars a barrel, how many barrels of cranberries can be bought for 770 dollars?

18. How many boxes will it require to pack 1530 pounds of butter, allowing 9 pounds to a box?

19. A man left 31265 dollars to be divided equally among his 5 children: how much did each receive?

20. A grocer sold oranges at 8 dollars a box, and received 22464 dollars: how many boxes did he sell?

21. If a man has 26436 acres of land, how many acres can he give to each of his 12 children?

22. A company of 11 men took a prize worth 116633 dollars, which was equally divided among them: what did each receive?

23. If a ship sails 10 miles an hour, how many hours will it be in sailing 25000 miles?

24. If 1 stage will seat 12 passengers, how many stages will be required to seat 1500 passengers?

LONG DIVISION.

1. Divide 22431 by 4, by Long Division.

OPERATION.

```
Div. Dividend.  Quot.
 4)22431(5607¾
   20'''
   —
   24
   24
   —
    031
     28
     —
      3 rem.
```

ANALYSIS.—Write the divisor on the left of the dividend, as in Short Division, and proceed thus: *First*, the divisor 4 is contained in 22, 5 times; set the 5 on the right of the dividend with a curve line between them. *Second*, multiply the divisor by this quotient figure, and set the product 20, under the figure divided. *Third*, subtract the product from the figures divided, and the remainder is 2. *Fourth*, bring down and annex to the remainder the next figure of the dividend, making 24 for the next partial dividend. Divide this partial dividend, and the quotient figure is 6. Multiply and subtract as before, and the remainder is 0. Bring down the next figure 3 for a new partial dividend. But the divisor 4 is not contained in 3; we therefore put a *cipher* in the quotient, and bringing down the next figure, we have 31 for a partial dividend, which we divide as before. As there are no more figures to be divided, we place the last remainder *over* the divisor, and *annex* it to the quotient. The answer is 5607¾.

NOTE.—To prevent mistakes, it is customary to place a *mark* under the several figures of the dividend, when brought down.

20. What is Long Division?

Long Division is the method of dividing when the *results* of the several steps and the *quotient* are both set down.

21. How write numbers for Long Division?

Place the *divisor* on the *left* of the dividend, and the *quotient* on the right, with a curve line between them.

22. How many steps in long division? "Four."

23. The first?

Find how many times the divisor is contained in the *fewest figures* on the *left* of the dividend that will contain it.

24. The second?

Multiply the divisor by the *quotient* figure, and set the *product* under the figures divided.

25. The third?

Subtract the product from the figures divided.

26. The fourth?

Annex to the remainder the *next figure* of the dividend, for a new *partial* dividend; then divide as before.

REMARK.—The quotient figure in *Long* and in *Short* Division, is the *same order* as the *right hand figure* of the partial dividend.

2. Divide 5463 by 4. *Ans.* $1365\frac{3}{4}$.

3. Divide 17382 by 5. 4. Divide 43652 by 6.

5. At 45 dollars an acre, how much land can be bought with 6750 dollars? *Ans.* 150 acres.

6. How many suits of clothes, at 63 dollars a suit, can be had for 7686 dollars? *Ans.* 122 suits.

27. The preceding principles may be summed up in the following

RULE FOR LONG DIVISION.

I. *Find how many times the divisor is contained in the fewest figures on the left of the dividend that will contain it, and set the quotient on the right.*

II. *Multiply the divisor by this quotient figure, and subtract the product from the figures divided.*

III. *To the right of the remainder, bring down the next figure of the dividend, and divide as before.*

IV. *If the divisor is not contained in a partial dividend, place a cipher in the quotient, bring down another figure, and thus continue the operation.*

If there is a remainder after dividing the last figure, set it over the divisor, and annex it to the quotient.

NOTES.—1. *Long* Division, is the same in principle as *Short*. The only difference is, in one the results of the several steps are carried in the *mind*, and in the other they are *set down*.

Short Division is the more *expeditious*, and should be employed when the divisor does not exceed 12.

2. If the *product* of the divisor into the figure placed in the quotient is *greater* than the partial dividend, it is plain the quotient figure is *too large*, and therefore must be *diminished*.

3. If the *remainder* is *equal* to or *greater* than the *divisor*, the quotient figure is *too small*, and must *be increased*.

EXAMPLES FOR PRACTICE.

1. How many times is 24 contained in 1963?

2. Divide 40369 by 18.

3. Divide 45683 by 21.

4. Divide 614897 by 35.

5. Divide 598061 by 47.

6. Divide 85345 by 53.

7. Divide 906530 by 68.

8. Divide 990046 by 74.

9. Divide 867604 by 84.

10. Required the quotient of 9134669 divided by 92.

11. How many cows, at 35 dollars apiece, can be bought for 7140 dollars?

12. How much land, at 28 dollars per acre, can be bought for 5611 dollars.

13. If a man earns 45 dollars a month, how long will it take him to earn 1620 dollars?

14. How many stoves, at 38 dollars each, can be bought for 6840 dollars?

15. If there is 1 year in 52 weeks, how many years are there in 8202 weeks?

16. If a man's expenses are 63 dollars a month, how long can he live on 5260 dollars?

17. If a man pay 70 dollars a hogshead for molasses, how many hogsheads can he buy for 6940 dollars?

18. At 87 dollars per yoke, how many yoke of oxen can be bought for 6525 dollars?

To find the *Quotient Figure*, when the Divisor is *large*.

19. Divide 12451 by 382.

ANALYSIS.—Taking 3 for a trial divisor, it is contained in 12, 4 times. But in multiplying the 8 by 4, we have 3 to carry, and 3 added to 4 times 3, make 15, which is larger than the partial dividend 12. Hence, 4 is too large for the quotient figure. We therefore place 3 in the quotient, and proceed as before.

```
382)12451(32
     1146
     ----
      991
      764
      ---
Rem.  227
```

28. How find the quotient figure, when the divisor is large?

Take the first figure of the divisor for a trial divisor, and find how many times it is contained in the first or first two figures of the dividend, making due allowance for carrying the tens of the product of the second figure of the divisor into the quotient figure.

20. Divide 8732409 by 657.   22. 10342675 ÷ 3435.

21. Divide 9753102 by 950.   23. 23046750 ÷ 7625.

24. A certain fort has provisions sufficient to last 1 man 15360 days: how long will it last 256 men?

25. The president's salary is 25000 dollars a year: how much is that per day?

26. At a certain auction, 498 pictures were sold for 13944 dollars: what was the average price?

CONTRACTIONS.

I. When the Divisor is 10, 100, 1000, etc.

1. At 100 dollars a set, how many sets of fur can be bought for 1935 dollars, and how much over?

ANALYSIS.—*Annexing* a cipher to a number, multiplies it by 10. (P. 56, Q. 17.)

Conversely, *removing* a cipher or figure from the *right* of a number, divides it by 10; for, each figure in the number is removed one place to the right. (P. 11, Q. 17.)

OPERATION.

```
1|00)19|35
```

Quot. 19, and 35 *Rem.*

In like manner, cutting off *two* figures from the right of a number, divides it by 100; cutting off *three*, by 1000, etc.

Now as the divisor is 100, it is only necessary to cut off two figures on the right of the dividend: those left, viz., 19, are the quotient, and those cut off, viz., 35, the remainder.

29. How proceed when the divisor is 10, 100, 1000, etc.?

From the right of the dividend cut off as many figures as there are ciphers in the divisor. The figures left will be the quotient; those cut off, the remainder.

2. Divide 8564 by 100.
3. Divide 46531 by 1000.
4. Divide 48000 by 1000.
5. Divide 4375681 by 10000.
6. 39467 by 10000.
7. 272364 by 100000.
8. 1000000 by 100000.
9. 85325764 by 1000000.

II. When there are Ciphers on the right of the Divisor.

10. At 20 dollars apiece, how many bureaus can be bought for 3453 dollars?

ANALYSIS.—The divisor, 20, is composed of the factors 2 and 10. In the operation, we first divide by 10, by cutting off the right-hand figure of the dividend; then divide the remaining figures by 2, the other factor of the divisor. The result 172 is the quotient; and 3, the figure cut off, being annexed to the remainder, forms the true remainder.

OPERATION.

$$2|0)\underline{345|3}$$

Ans. 172 b. 13 rem.

30. How proceed, when there are ciphers on the right of the divisor?

I. *Cut off the ciphers on the right of the divisor, and as many figures on the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor for the quotient.*

III. *Annex the figures cut off to the remainder, and the result will be the true remainder.*

11. Divide 8534 by 20.
12. Divide 12345 by 30.
13. 163045 ÷ 1900.
14. Divide 23681 by 300.
15. Divide 40642 by 1300.
16. 264168 ÷ 31000.

DRILL FOR RAPID COMBINATIONS.

To Teachers.—These and other drill exercises, should be continued but a few minutes at a time. If *spirited* and *frequent*, better results will be obtained from them, though short, than from scores of examples recited in an *indifferent*, *sluggish* manner.

Oral.—1. To 4 add 8; subtract 2; multiply by 3; divide by 5; add 4; multiply by 3; add 10; result?

2. From 12 subtract 5; add 2; multiply by 4; divide by 6; add 5; multiply by 3; result?

3. Multiply 3 by 6; add 4; subtract 2; divide by 5; multiply by 6; add 8; divide by 4; result?

4. Divide 42 by 7; multiply by 4; subtract 6; divide by by 3; add 5; add 9; divide by 5; multiply by 11; result?

5. To 14 add 8; take 4; divide by 9; multiply by 8; add 8; divide by 8; multiply by 9; result?

6. From 27 take 9; divide by 9; multiply by 9; add 9; take 7; multiply by 2; divide by 10; result?

7. Multiply 9 by 7; subtract 7; divide by 8; add 12; subtract 4; divide by 5; multiply by 12; divide by 9; add 20; divide by 6, multiply by 11; result?

8. Divide 54 by 9; multiply by 7; subtract 6; divide by 9; multiply by 8; add 7; subtract 4; divide by 7; add 30; result?

9. Add 7 to 15; divide by 11; multiply by 9; add 10; divide by 7; multiply by 12; add 11; subtract 4; divide by 5; multiply by 8; result?

10. Multiply 8 by 7; subtract 6; divide by 10; multiply by 9; add 11; divide by 8; multiply by 9; subtract 3; add 30; subtract 7; result?

Slate.—1. To 36 add 45; subtract 37; multiply by 6; divide by 8; multiply by 9; add 99; divide by 9; add 200; result?

2. From 87 take 33; multiply by 7; divide by 6; add 233; take 48; divide by 8; multiply by 25; result?

3. Multiply 348 by 9; add 556; divide by 8; multiply by 48; divide by 24; add 545; take 378; result?

4. Divide 576 by 24; multiply by 35; add 1200; divide by 20; multiply by 45; divide by 9; take 375; add 2375; result?

5. To 785 add 357; take 571; add 629; divide by 24; multiply by 64; divide by 32; add 873; take 367; result?

6. From 3256 take 840; divide by 302; add 78; multiply by 56; divide by 28; add 1575; result?

7. Multiply 456 by 28; divide by 7; add 256; take 1200; divide by 44; multiply by 325; result?

QUESTIONS FOR REVIEW.

ORAL.—1. If 1 man can do a job of work in 72 days, how long will it take 9 men to do it?

ANALYSIS.—9 men can do 9 days work in 1 day: therefore, to do 72 days work, it will take them as many days as 9 is contained times in 72, which is 8. *Ans.* 8 days.

2. If a barrel of apples will last 1 person 56 days, how long will it last a family of 7 persons?

3. How many weeks in 8 times 9 days?

4. How many 4-quart cans can be filled from three 8-quart pails?

5. A farmer bought 4 pair of boots, at 5 dollars a pair, and paid for them in wheat, at 2 dollars a bushel: how many bushels did the boots come to?

6. How many times 8 in 7 times 12?

7. A market-woman sold 6 dozen eggs, at 10 cents a dozen, and took her pay in muslin, at 12 cents a yard: how many yards did she receive?

8. How many 2-gallon measures can be filled from 6 ten-gallon casks of water?

9. Herbert bought 20 marbles at one time, and 16 at another; meantime he lost 12: how many had he then?

10. If you earn 9 dollars a week, and pay 3 dollars for board, and 2 dollars for incidentals, how much will you lay up in 9 weeks?

11. In 40 less 12, how many times 7?

12. A trader bought 12 pair of shoes at 2 dollars, and 6 hats at 5 dollars: how much did he pay for both?

13. Three men gave a poor person 75 dollars; one gave 30 dollars, and another 25 dollars: how much did the other give?

14. Three lads, counting their money, found A had 25 cents, B twice as much as A, and C as much as both the others: how much had all?

SLATE.—1. A fort has provisions sufficient to last 1 man 365 days: how long will it last a company of 73 men?

2. Bought 100 hogs, weighing 300 pounds each, at 7 cents a pound, and sold them at 10 cents a pound: what was the profit?

3. A grocer bought 15 hogsheads of molasses, at 35 dollars per hogshead; 151 boxes of oranges, at 6 dollars a box; and 91 sacks of coffee, at 20 dollars a sack; and sold the whole for 4856 dollars: what did he make by the operation?

4. A farmer having 115 dollars, paid 40 dollars for a cow, and the remainder for 15 sheep: what did the sheep cost him apiece?

5. A shoe-dealer sold 87 pair of overshoes, at 2 dollars a pair; 110 pair of boots, at 9 dollars a pair; and took his pay in coal, at 11 dollars a ton: how much coal ought he to receive?

6. A teacher was engaged at 1260 dollars a year; at

the end of 9 months his health failed and he left: how much should he receive?

7. A grocer bought 455 barrels of flour for 3185 dollars; he afterward bought another lot at the same rate for 1610 dollars: how many barrels were there in both lots; and what did it cost him per barrel?

8. A man left 6528 dollars to his wife and 3 children; to the latter he gave 1265 dollars apiece: what was the portion of his wife?

9. A young man's salary amounted to 1208 dollars a year for 3 years; his expenses the first year were 375 dollars; the second, 420 dollars; and the third, 519 dollars: how much did he lay up in the 3 years?

10. If I earn 1350 dollars a year, and spend 1785 dollars a year, how much shall I be in debt in 3 years?

11. What number taken from 25973 + 8230 will leave 8768?

12. What number taken from 41260 — 3281 will leave 8600?

13. What number taken from 62135 — 1612 will leave 21500 — 2861?

14. If one man can perform a piece of work in 750 days, how long will it take 25 men to do it?

15. If a man earns 1645 dollars a year, and his expenses are 517 dollars a year: how long will it take him to lay up 4512 dollars?

16. Three lads, talking of their money, the first said he had 187 cents; the second said he had as much as the first minus 23 cents; and the third said if he had 40 cents more, he should have as many as the other two: how many cents had the second? The third?

17. Two men being 1950 miles apart, traveled towards each other at the rate of 35 and 43 miles a day respectively: how long before they met?

FACTORING.

*** Teachers who prefer to have pupils study United States Money before Common and Decimal Fractions, are referred to p. 143.

1. What two numbers multiplied together make 6?
2. What then are the factors of 6? (P. 47, Q. 5.)
3. What are the factors of 10?
4. What are the factors of 8? Of 12?
5. What are the factors of 14? Of 15? Of 21?
6. Name two factors of 16. Of 18. Of 20.
7. Name two factors of 24. Of 35. Of 48.
8. Name two factors of 54. Of 63. Of 72.
9. Name two factors of 84. Of 96. Of 108.

DEFINITIONS.

1. What is a Factor?

A ***Factor*** of a number is one of the numbers, which multiplied together, produce that number. (P. 47, Q. 5.)

2. What is a Composite Number?

A ***Composite Number*** is the *product* of two or more *factors*, each of which is *greater* than 1. Thus, when it is said that $3 \times 5 = 15$, fifteen is a composite number, and 3 and 5 are its factors.

3. What is a Prime Number?

A ***Prime Number*** is one which *cannot* be produced by multiplying any two numbers together, except *unit* and *itself.*

4. What are Prime Factors?

The ***Prime Factors*** of a number are the prime numbers which, multiplied together, produce that number.

5. What is an Odd Number?

An ***Odd Number*** is one which *cannot* be divided by 2, without a remainder; as, 1, 3, 5, 7, etc.

6. What is an Even Number?

An ***Even Number*** is one which *can* be divided by 2, without a remainder; as, 2, 4, 6, 8, etc.

NOTE.—All *even* numbers except 2 are *composite* numbers.

7. What is meant by Factoring a number?

Factoring a Number is finding two or more *factors* which multiplied together, produce that number.

MENTAL EXERCISES.

1. Name the odd numbers under 30.
2. Name the even numbers under 30.
3. Name all the composite numbers under 30.
4. Name all the prime numbers under 30.
5. What are the prime factors of 30?

ANALYSIS.—By inspection we perceive that 30 is divisible by the prime number 2, giving the factors 2 and 15. Again, dividing 15 by 3 we have the factors 3 and 5, both of which are prime Therefore, 2, 3, and 5, are the prime factors required.

6. What are the prime factors of 12? 15? 18?
7. What are the prime factors of 20? 28? 30?
8. What are the prime factors of 35? 40? 42?

SLATE EXERCISES.

1. What are the prime factors of 105?

ANALYSIS.—Dividing 105 by the prime number 3, we have the factors 3 and 35. Again, dividing the 1st quotient 35, by the prime number 5, we have the factors 5 and 7. Finally, dividing the 2d quotient 7 by 7, we have 7 and 1. But the divisors 3, 5, and 7 are all prime numbers, and therefore are the prime factors required.

OPERATION.

1st divisor, 3	105, given.
2d " 5	35, 1st quot.
3d " 7	7, 2d "
$105 = 3 \times 5 \times 7$	1, 3d "

8. How is a composite number resolved into *prime factors?*

Divide the given number by any prime number that will divide it without a remainder. Again, divide this quotient by a prime number, and so on till the quotient is 1. *The several divisors are the prime factors required.*

NOTE.—The *least divisor* of every number is a *prime factor;* hence, to avoid mistakes, it is advisable for beginners to take for the divisor, the *least number* that will divide the several dividends *without a remainder.*

Find the prime factors of the following numbers:

2. 42.	6. 100.	10. 200.	14. 625.
3. 48.	7. 125.	11. 256.	15. 1000.
4. 60.	8. 132.	12. 325.	16. 1728.
5. 72.	9. 175.	13. 450.	17. 1872.

CANCELLATION.

1. What is the quotient of $3 \times 3 \times 5$ divided by 3×5?

ANALYSIS.—$3 \times 3 \times 5 = 45$, and $3 \times 5 = 15$; now $45 \div 15 = 3$. But it will be seen by inspection that the factors 3 and 5 are common tc the dividend and the divisor. If we cancel or cross out the 3 in each, we have $3 \times 5 \div 5$, or $15 \div 5$, which equals 3, the same as before. Again, if we cancel or cross out the 5 in each, we have $3 \div 1$, which equals 3, as before.

NOTE.—To *cancel* a factor of a number means to *erase* or *reject* it.

2. What is the quotient of $2 \times 3 \times 7 \div 2 \times 3 \times 5$?

SOLUTION.—Cancelling the common factors 2 and 3, we have $7 \div 5 = 1\frac{2}{5}$ *Ans.*

1. What is the effect of cancelling a factor from a number?

It ***divides the number*** by that factor.

10. What is Cancellation?

Cancellation is the method of *abbreviating operations* by *rejecting equal factors* from the divisor and dividend.

REMARK.—When the *factor cancelled* is equal to the *number* itself, 1 is always left in its place; for, dividing a number by itself, the quotient is 1. When the 1 stands in the dividend, it must be retained; when in the divisor, it may be disregarded.

3. What is the quotient of $2 \times 5 \times 7 \div 2 \times 3 \times 7$?

ANALYSIS.—Writing the divisor under the dividend, and cancelling the factors 2 and 7, which are common to both, we have $1 \times 5 \times 1 \div 1 \times 3 \times 1$. Now the product of $1 \times 5 \times 1 = 5$, that of $1 \times 3 \times 1 = 3$, and $5 \div 3 = 1\frac{2}{3}$ *Ans.*

OPERATION.

$$\frac{\overset{1}{\cancel{2}} \times 5 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 3 \times \underset{1}{\cancel{7}}} = \frac{1 \times 5 \times 1}{1 \times 3 \times 1} = \frac{5}{3}, \text{ or } 1\frac{2}{3}$$

11. What is the rule for Cancellation?

Cancel all the factors common to the divisor and dividend, and divide the product of those remaining in the dividend by the product of those remaining in the divisor.

4. What is the quotient of 77 divided by 21?

SOLUTION.—By inspection, we perceive that 7 is a factor common to the divisor and the dividend. Cancelling this factor, we have $\frac{11}{3}$, or $3\frac{2}{3}$, *Ans.*

OPERATION.

$$\frac{\cancel{77},}{\cancel{21},} \quad \frac{11}{3} = 11 \div 3, \text{ or } 3\frac{2}{3}$$

Perform the following divisions by cancellation.

5. $4 \times 5 \times 7 \div 5 \times 4 \times 3$.
6. $7 \times 3 \times 11 \div 8 \times 3 \times 7$.
7. $23 \times 5 \times 9 \div 5 \times 7 \times 9$.
8. $28 \times 13 \times 11 \div 11 \times 13 \times 7$.
9. $63 \times 39 \times 2 \div 13 \times 9 \times 3$.
10. $96 \times 7 \times 11 \div 12 \times 8 \times 7$.

11. How many yards of cloth, at 8 dollars a yard, can be bought for 25 pair of boots, at 4 dollars a pair?

12. How many barrels of flour, at 7 dollars a barrel, must be given for 18 tons of hay, at 14 dollars a ton?

13. How long must a man work, at 3 dollars a day to pay his rent for a year, at 11 dollars a month?

COMMON DIVISORS.

MENTAL EXERCISES.

1. What will divide 9 and 15 without a remainder?
2. What will divide 14 and 24 without a remainder?
3. What will divide 16 and 20 without a remainder?
4. What will divide 42 and 18 without a remainder?
5. What is the greatest divisor of 18 and 27?
6. What is the greatest divisor of 12 and 36?

DEFINITIONS.

12. What is a Common Divisor?

A ***Common Divisor*** is a number which will divide *two* or *more* numbers without a *remainder.*

13. What is the Greatest Common Divisor?

The ***Greatest Common Divisor*** of two or more numbers, is the *greatest number* that will divide each of them without a *remainder.*

REMARKS.—1. A *common divisor* of two or more numbers is always a *common factor* of those numbers; and the *greatest common* divisor of them is their *greatest common* factor.

2. A *common divisor* is often called a *common measure.*

3. The *greatest common divisor* of two or more numbers is equal to the *product* of all the *prime factors common* to those numbers.

1. What is the greatest common divisor of 16 and 28?

1ST METHOD.—Dividing the *greater* by the *less,* the quotient is 1, and 12 remainder. Again, dividing the *first divisor* by the *first* remainder 12, the quotient is 1, and 4 remainder. Next, dividing the *second divisor* by the *second* remainder 4, the quotient is 3, and 0 remainder. The last divisor 4, is the *greatest common* divisor.

1ST OPERATION.

$$\begin{array}{l} 16)\,28\,(1 \\ \quad\;\; \underline{16} \\ \quad\;\; 12)\,16\,(1 \\ \qquad\quad\;\; \underline{12} \\ \qquad\quad\;\; 4)\,12\,(3 \\ \qquad\qquad\;\; \underline{12} \end{array}$$

2D METHOD.—Setting the numbers in a *horizontal* line, divide by any prime number, as 2, that will divide each of them without a remainder, and set the quotients under the corresponding numbers. Dividing each of these quotients by 2 again, the new quotients 4 and 7 have no common factor. Hence, the product of the common divisors 2 into 2, or 4, is the greatest common divisor.

2D OPERATION.

2)	16	28
2)	8	14
	4	7

Ans. $2 \times 2 = 4$.

14. How find the greatest common divisor of two or more numbers?

Divide the greater number by the less, the first divisor by the first remainder, the second divisor by the second remainder, and so on until the remainder is nothing; the last divisor will be the greatest common divisor.

Or, write the numbers in a horizontal line, and divide by any prime number that will divide each without a remainder; setting the quotients in a line below.

Divide these quotients as before, and thus proceed, till no number can be found that will divide all the quotients without a remainder. The product of all the divisors will be the greatest common divisor.

NOTES.—1. If there are *more* than *two* numbers, and the first method is used, first find the greatest common divisor of two of them, then of this divisor and a third number, and so on, until all the numbers have been used.

2. When there are *three* or *more* numbers, the second method has the advantage both in *simplicity* and *facility* of application.

SLATE EXERCISES.

2. Required the greatest com. divisor of 15, 45, and 60?

SOLUTION.—Divide the given numbers by 3, and the quotients thence arising by 5; the next quotients, 1, 3, and 4, have no common factor. Hence, the product of 3 into 5, or 15, is the answer.

3)	15	45	60
5)	5	15	20
	1	3	4

$3 \times 5 = 15$, *Ans.*

Find the greatest common divisor of the following numbers:

2. 27 and 36.
3. 32 and 48.
4. 45 and 60.
5. 72 and 24.
6. 75 and 105.
7. 81 and 108.
8. 120 and 148.
9. 256 and 512.
10. 36, 84, and 108.
11. 45, 60, and 135.
12. 30, 75, and 225.
13. 48, 144, and 288.

14. What is the greatest number by which 128, 160, and 192 can be exactly divided?

15. What is the longest pole by which 108, 132, and 144 feet can be exactly measured?

16. A shopkeeper has three balls of twine, containing 120, 100, and 200 yards, which he wishes to cut into kite-lines of equal length: what is the greatest length he can make them?

COMMON MULTIPLES.

MENTAL EXERCISES.

1. What numbers under 12 can be divided by 2 without a remainder?

2. By what numbers can 12 be exactly divided?

3. What numbers under 20 can be exactly divided by 4?

4. By what numbers can 15 be exactly divided?

5. By how many numbers can 18 be exactly divided?

6. By what numbers can 24 be exactly divided?

7. By what two factors can 33 be exactly divided?

8. By what two factors can 35 be exactly divided?

9. What 4 numbers will exactly divide 42?

10. Name two numbers that can be exactly divided by 4, 5, and 6.

DEFINITIONS.

15. What is a Multiple?

A ***Multiple*** is a number which can be divided by another number *without a remainder*.

16. What is a Common Multiple?

A ***Common Multiple*** is a number which can be divided by *two* or *more* numbers without a remainder. Thus, 15 is a common multiple of 3 and 5.

REMARK.—A common multiple of two or more numbers, contains *all the prime factors* of those numbers.

17. What is the Least Com. Multiple of two or more numbers?

The ***Least Common Multiple*** of two or more numbers, is the *least* number which can be divided by each of them without a remainder. Thus, 12 is the least common multiple of 2, 3, and 4.

1. What is the least com. multiple of 12, 18, and 21?

1ST OPERATION.

$$\begin{array}{r|rrr} 3 & 12 & 18 & 21 \\ \hline 2 & 4 & 6 & 7 \\ \hline & 2 & 3 & 7 \end{array}$$

$$3 \times 2 \times 2 \times 3 \times 7 = 252$$

1ST METHOD.—Writing the numbers in a horizontal line, we divide by any prime number 3, which will divide two or more of them without a remainder, and set the quotients in the line below. Again, dividing these quotients by the prime number 2, which will divide two of them without a remainder, we set the quotients and undivided number 7 in a line below as before. As the numbers 2, 3, and 7, are prime factors, the division can be carried no further. Finally, the continued product of the divisors and numbers in the last line, $3 \times 2 \times 2 \times 3 \times 7 = 252$, is the least com. multiple.

2D OPERATION.

$$12 = 2 \times 2 \times 3$$
$$18 = \cancel{2} \times \cancel{3} \times 3$$
$$21 = \cancel{3} \times 7$$
$$2 \times 2 \times 3 \times 3 \times 7 = 252$$

2D METHOD.—Resolve the given numbers into their prime factors, as in the margin. But we have seen that a common multiple of two or more numbers contains all the prime factors of those numbers. Hence, it must contain the prime factors of 12, which are $2 \times 2 \times 3$; we therefore retain these factors. Again, it must contain the prime factors of 18,

which are $2 \times 3 \times 3$. But we already have two 2s and one 3; we may therefore cancel the 2, and one of the 3s, retaining the other 3. Finally, it must contain the factors of 21, which are 3×7. But since we have retained two 3s, we may cancel this 3, and retain the 7. The continued product of the *uncancelled factors* $2 \times 2 \times 3 \times 3 \times 7 = 252$, the same as before.

18. How find the least common multiple of two or more numbers?

Write the numbers in a horizontal line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and numbers undivided in a line below.

Next divide this line as before, and thus proceed till no two numbers are divisible by any number greater than 1. The continued product of the divisors and numbers in the last line will be the answer.

Or, resolve the given numbers into their prime factors; multiply these factors together, taking each the greatest number of times it occurs in either of the given numbers, and the product will be the answer.

REMARK.—Both of these methods are based upon the principle, that the *least common multiple* of two or more numbers is the least number which contains *all their prime factors*, each factor being taken as *many times* as it occurs in either of the given numbers.

SLATE EXERCISES.

Find the least common multiple of the following:

2. 4, 8, 12.
3. 16, 12, 24.
4. 15, 30, 45.
5. 36, 48, 84.
6. 40, 45, 75.
7. 20, 60, 55.
8. 39, 52, 13.
9. 81, 108, 72.
10. 24, 12, 48, 60.
11. 14, 42, 28, 56.
12. 54, 81, 96, 120.
13. 72, 144, 288, 432.

FRACTIONS.

INTRODUCTORY EXERCISES.

To Teachers.—The object of this Exercise is to develop the idea of *Fractional parts*. The best way to secure this end is, to let beginners divide some object, as a sheet of paper, or an apple, into *halves*, *thirds*, *fourths*, etc.; then put the parts together and form the *whole* again.

1. If you divide a sheet of paper into *two equal* parts, what is each part called?

One half.

2. Draw a line an inch long upon your slate or blackboard, and divide it into *halves*.

3. If you divide an apple into *three equal* parts, what is one of the parts called?

One third.

4. Two of the parts?

Two thirds.

5. Into how many *halves* can you divide an apple? Into how many *thirds?*

6. Draw a line a foot long, and divide it into *halves*. Into *thirds*.

7. How many thirds make a whole one?

8. If a sheet of paper is divided into *four equal* parts, what is one of the parts called?

A Fourth, or *quarter*.

9. What are two of the parts called? Three of the parts? How many fourths in a whole sheet?

10. When a thing is divided in *five equal* parts, what are the parts called?

Fifths.

11. When a thing is divided in *six equal* parts, what are the parts called? If divided in seven, what? Into eight, what? Into nine, what? Into ten, what? Into twenty, what? Into fifty, what?

12. Which are greater, halves or thirds? Thirds or fourths? Sixths or fifths? Tenths or eighths?

DEFINITIONS.

1. What is an Integer?

An ***Integer*** is a number which contains *one* or *more entire* units only; as 1, 3, 5, 8, 12, etc.

2. What is a Fraction?

A ***Fraction*** is one or more of the *equal parts* into which a *unit* is divided.

3. What is meant by one-half?

One of the *two equal parts* into which a unit is divided.

What is meant by a third? Two thirds? A fourth? Fifth? Seventh? Tenth?

4. From what do these parts take their name?

The ***Number of equal parts*** into which the *unit* or *thing* is divided.

5. Upon what does their value depend?

First. Upon the ***magnitude*** of the *unit* or *thing* divided.

Second. Upon the ***number of parts*** into which it is divided.

Illustrate these two points:

1st. If a *large* and a *small* sheet of paper are each divided into *halves, thirds, fourths,* etc., it is plain that the parts of the *former* will be larger than the corresponding parts of the *latter.*

2d. If *one* of *two equal* sheets of paper is divided into *two* equal parts, and the *other* into *four*, the parts of the first will be *twice* as *large* as those of the second; if one is divided into *two* equal parts, the other into *six*, one part of the *first* will be equal to *three* of the *second*, etc. Hence,

NOTE.—1. A *half* is *twice* as large as a *fourth, three times* as large as a *sixth, four times* as large as an *eighth*, etc.; and generally,

2. The *greater* the *number* of equal parts into which the unit is divided, the *less* will be the *value* of each part. Conversely,

3. The *less* the *number* of equal parts, the *greater* will be the value of each part.

FINDING FRACTIONAL PARTS.

1. What is 1 half of 10 dollars?

ANALYSIS.—If 10 dollars are divided into *two equal* parts, *one* o. these parts is 5 dollars. Therefore, &c. (P. 63, Q. 11.)

2. What is 1 half of 8 peaches?
3. What is a third of 9? Of 15? Of 18? Of 24?
4. What is a fourth of 12? Of 16? Of 28? Of 40?
5. What is a fifth of 20? Of 45? Of 35? Of 60?
6. What is an eighth of 32? Of 56? Of 64? Of 72?
7. What is 2 thirds of 18 yards?

ANALYSIS.—2 thirds are twice 1 third; now 1 third of 18 yards is 6 yards, and 2 times 6 yards are 12 yards. Therefore, etc.

8. What is 3 fourths of 32?
9. What is 3 eighths of 48?
10. What is 4 fifths of 35?
11. What is 7 tenths of 60?

NOTATION OF FRACTIONS.

6. Into what *two classes* are fractions divided?

Into ***Common*** and ***Decimal.***

7. What is a Common Fraction?

A ***Common Fraction*** is one in which the unit is divided into *any number* of equal parts.

8. How are common fractions usually expressed?

By ***Figures*** written above and below a line, called the *numerator* and *denominator;* as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{12}$.

9. **Where is the Denominator placed, and what does it show?**

The ***Denominator*** is written *below* the line, and shows into *how many equal parts* the unit is divided.

10. **Where the Numerator, and what does it show?**

The ***Numerator*** is written *above* the line, and shows *how many parts* are expressed by the fraction.

NOTES.—1. The denominator is so called because it *names* the parts; as, halves, thirds, etc.

2. The numerator is so called because it *numbers* the parts taken. Thus, in the fraction $\frac{2}{3}$, 3 is the denominator, and shows that the unit is divided into 3 equal parts; 2 is the numerator, and shows that 2 of the parts are taken.

11. **What are the Terms of a fraction?**

The ***Terms of a Fraction*** are the *Numerator* and *Denominator*.

WRITTEN EXERCISES.

1. How express one-half, one-third, three-fourths, etc., by figures? *Ans.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$.

Write the following fractions in figures:

2. Two thirds.
3. Four fifths.
4. Three sevenths.
5. Seven eighths.
6. Five ninths.
7. Seven thirds.
8. Four fourths.
9. Ten twelfths.
10. Eight twenty-thirds.
11. Nine thirty-firsts.
12. Twenty-three fortieths.
13. Nineteen seventy-fifths.
14. Seventy-four hundredths.
15. Ninety-nine thousandths.

Copy and read the following:

16. $\frac{5}{8}$.	21. $\frac{10}{21}$.	26. $\frac{21}{13}$.	31. $\frac{135}{278}$.
17. $\frac{7}{12}$.	22. $\frac{12}{30}$.	27. $\frac{35}{7}$.	32. $\frac{347}{463}$.
18. $\frac{8}{15}$.	23. $\frac{13}{49}$.	28. $\frac{43}{8}$.	33. $\frac{407}{360}$.
19. $\frac{9}{16}$.	24. $\frac{65}{78}$.	29. $\frac{61}{5}$.	34. $\frac{580}{945}$.
20. $\frac{7}{19}$.	25. $\frac{74}{93}$.	30. $\frac{59}{7}$.	35. $\frac{800}{1000}$.

DEFINITIONS.

12. **Into what are common fractions divided?**

Into proper, improper, simple, compound, complex fractions, and mixed numbers.

13. **Explain each.**

A ***Proper Fraction*** is one whose numerator is *less* than the denominator; as, $\frac{1}{2}$, $\frac{3}{4}$.

An ***Improper Fraction*** is one whose numerator *equals* or *exceeds* the denominator; as, $\frac{4}{4}$, $\frac{5}{3}$.

A ***Simple Fraction*** is one having but *one* numerator and *one* denominator, each of which is a whole number, and may be *proper* or *improper;* as, $\frac{2}{3}$, $\frac{5}{4}$.

A ***Compound Fraction*** is a *fraction* of a fraction; as $\frac{1}{3}$ of $\frac{3}{4}$.

A ***Complex Fraction*** is one which has a *fractional* numerator, and an *integral* denominator: as, $\frac{\frac{1}{2}}{3}$, $\frac{2\frac{2}{3}}{4}$.*

A ***Mixed Number*** is a *whole* number and a *fraction* expressed together; as, $5\frac{3}{7}$, $34\frac{13}{23}$.

1. What kind of fractions are $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{7}{8}$? Why?
2. What kind are $\frac{5}{5}$ and $\frac{9}{4}$? $\frac{6}{3}$ and $\frac{9}{9}$? Why?
3. What kind are $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{4}{7}$ of $\frac{5}{8}$? Why?
4. What do you call $4\frac{1}{2}$, $7\frac{3}{5}$, $9\frac{2}{3}$? Why?
5. What do you call $\frac{3\frac{1}{2}}{4}$, $\frac{\frac{2}{3}}{5}$? Why?

14. **What is the value of a fraction?**

The ***Value of a Fraction*** is the *quotient* of the numerator divided by the denominator. Thus, the value of 1 half is $1 \div 2$; of 2 thirds, is $2 \div 3$; of 4 fourths, is $4 \div 4$, or 1; of 6 thirds, is $6 \div 3$, or 2, etc.

* See New Practical Arithmetic, Rem. p. 101.

GENERAL PRINCIPLES OF FRACTIONS.

15. **What are some of the principles upon which the operations in fractions depend?**

I. *Multiplying the numerator by any number, multiplies the fraction by that number.*

II. *Dividing the numerator, divides the fraction.*

III. *Multiplying the denominator, divides the fraction.*

IV. *Dividing the denominator, multiplies the fraction.*

V. *Multiplying or dividing both the numerator and denominator by the same number, does not alter the value of the fraction.*

REDUCTION OF FRACTIONS.

MENTAL EXERCISES.

1. If I divide an apple into halves, how can I express one of these parts by figures?

By $\frac{1}{2}$. (The pupil writes it upon the blackboard.)

2. If you multiply both terms of $\frac{1}{2}$ by 2, what will it become?

It will become $\frac{2}{4}$.

3. If you multiply both terms of $\frac{1}{2}$ by 3, 4, 5, etc., what?

It will become $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, and so on.

4. How many fourths in $\frac{1}{2}$?

Two fourths.

5. How many sixths in $\frac{1}{2}$? How many eighths? How many tenths? Twelfths?

6. If you multiply both terms of $\frac{1}{3}$ by 2, what will it become?

It will become $\frac{2}{6}$.

7. If you multiply both terms of $\frac{1}{3}$ by 3, 4, 5, etc., what will it become?

It will become $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$, etc.

8. How many thirds in $\frac{2}{6}$? In $\frac{4}{12}$? $\frac{6}{18}$? $\frac{10}{30}$?

9. If I divide an orange into 4 equal parts, how can I express one-half of these parts by figures?

By $\frac{2}{4}$. (The pupil writes it upon the blackboard.)

10. If you divide both terms of $\frac{2}{4}$ by 2, what will it become?

It will become $\frac{1}{2}$.

11. To how many halves are $\frac{4}{8}$ equal? $\frac{5}{10}$? $\frac{6}{12}$?

12. To how many thirds are $\frac{4}{6}$ equal? $\frac{6}{9}$? $\frac{8}{12}$?

13. How many fourths equal $\frac{4}{8}$? $\frac{6}{12}$? $\frac{12}{16}$?

DEFINITIONS.

16. ***What is Reduction of Fractions?***

Reduction of Fractions is *changing the terms,* without *altering the value* of the fractions.

17. **What is meant by reducing a fraction to *higher* terms?**

It is changing its numerator and denominator to *larger numbers,* without *altering* its *value.*

18. **What by reducing a fraction to *lower* terms?**

It is changing its numerator and denominator to *smaller numbers,* without *altering* its *value.*

19. **What is the principle upon which these changes are made?**

The principle that *multiplying or dividing* both the numerator and denominator by the *same number* does not alter the *value* of the fraction. (P. 93, Pr. V.)

CASE I.

To reduce a Fraction to *higher* terms.

1. Reduce $\frac{3}{5}$ to twentieths.

ANALYSIS.—The required denominator 20, contains 5, the given denominator, 4 times. But if both terms of a fraction are multiplied by the same number, its value is not altered. Therefore, multiplying both terms of $\frac{3}{5}$ by 4, we have $\frac{3}{5}=\frac{12}{20}$, the fraction required. (P. 93, Prin. V.)

OPERATION.

$20 \div 5 = 4.$

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}, \textit{ Ans.}$$

20. How reduce a fraction to higher terms?

Multiply both terms of the fraction by such a number as will make the given denominator equal to the required denominator.

NOTE.—The *multiplier* is found by *dividing the proposed denominator* by the *given* denominator.

2. Reduce $\frac{4}{5}$ to thirtieths.
3. Reduce $\frac{5}{8}$ to fortieths.
4. Reduce $\frac{7}{9}$ to sixty-thirds.
5. Reduce $\frac{5}{11}$ to fifty-fifths.
6. Reduce $\frac{9}{15}$ to seventy-fifths.
7. Reduce $\frac{3}{19}$ to seventy-sixths.
8. Reduce $\frac{11}{45}$ to one-hundred-and-thirty-fifths.
9. Reduce $\frac{23}{67}$ to two-hundred-and-sixty-eighths.
10. Reduce $\frac{97}{125}$ to one-thousandths.

CASE II.

To reduce a Fraction to *lower* terms.

11. To how many tenths are $\frac{12}{20}$ equal?

OPERATION.

$$20 \div 10 = 2.$$

$$\frac{12 \div 2}{20 \div 2} = \frac{6}{10}, \textit{ Ans.}$$

ANALYSIS.—The given denominator 20 contains 10, the required denominator, 2 times. But, if both terms of a fraction are divided by the same number, its value is not altered; therefore, dividing both terms of $\frac{12}{20}$ by 2, it becomes $\frac{6}{10}$, the fraction required. (P. 93, Prin. V.)

21. How is a fraction reduced to lower terms?

Divide both terms by such a number as will make the given denominator equal to the required denominator.

12. Reduce $\frac{6}{12}$ to fourths.
13. Reduce $\frac{5}{15}$ to thirds.
14. Reduce $\frac{4}{16}$ to eighths.
15. Reduce $\frac{9}{18}$ to sixths.
16. Reduce $\frac{12}{24}$ to fourths.
17. Reduce $\frac{7}{35}$ to fifths.
18. Reduce $\frac{18}{45}$ to ninths.
19. Reduce $\frac{36}{54}$ to sixths.
20. Reduce $\frac{63}{72}$ to eighths.
21. Reduce $\frac{84}{108}$ to twelfths.

CASE III.

To reduce a Fraction to its *lowest* terms.

22. What are the *lowest* terms of a fraction?

The ***Lowest Terms*** of a fraction are the *smallest numbers* in which its numerator and denominator can be expressed.

1. What are the lowest terms to which $\frac{12}{16}$ can be reduced?

ANALYSIS.—Dividing both terms of a fraction by the *same number*, does not alter its value. (Prin. V.) Now if we divide both terms of $\frac{12}{16}$ by 2, we have $\frac{6}{8}$. Again, dividing both terms of the new fraction $\frac{6}{8}$ by 2, the result is $\frac{3}{4}$, which are the lowest terms in which $\frac{12}{16}$ can be expressed.

1ST OPERATION.

$2)\frac{12}{16}=\frac{6}{8}$ and

$2)\frac{6}{8}=\frac{3}{4}$, *Ans.*

Or, we may divide both terms of the fraction by their *greatest* common divisor, which is 4, and obtain the same result. (P. 84, Q. 14.)

2D OPERATION.

$4)\frac{12}{16}=\frac{3}{4}$, *Ans.*

23. How reduce a fraction to its *lowest terms?*

Divide the numerator and denominator continually by any number that will divide both without a remainder, until no number greater than 1 *will divide them.*

Or, divide both terms of the fraction by their greatest common divisor. (P. 84, Q. 14.)

2. What are the lowest terms to which $\frac{16}{20}$ can be reduced? *Ans.* $\frac{4}{5}$.

Reduce the following fractions to their lowest terms:

3. $\frac{20}{30}$.	9. $\frac{42}{72}$.	15. $\frac{37}{93}$.	21. $\frac{91}{189}$.
4. $\frac{24}{36}$.	10. $\frac{64}{96}$.	16. $\frac{61}{103}$.	22. $\frac{284}{412}$.
5. $\frac{18}{36}$.	11. $\frac{84}{32}$.	17. $\frac{108}{324}$.	23. $\frac{125}{300}$.
6. $\frac{25}{45}$.	12. $\frac{65}{150}$.	18. $\frac{64}{100}$.	24. $\frac{121}{363}$.
7. $\frac{42}{63}$.	13. $\frac{96}{144}$.	19. $\frac{81}{243}$.	25. $\frac{250}{750}$.
8. $\frac{54}{81}$.	14. $\frac{108}{156}$.	20. $\frac{46}{184}$.	26. $\frac{375}{1025}$.

CASE IV.

To reduce an Improper Fraction to a *Whole* or *Mixed Number*.

1. In 7 half dimes, how many whole dimes?

ANALYSIS.—Since in 2 halves there is 1 whole dime, in 7 halves there are as many dimes as 2 is contained times in 7, which is 3 and 1 half over, or $3\frac{1}{2}$ times. Therefore, in 7 half dimes there are $3\frac{1}{2}$ dimes.

2. In 10 half dollars, how many dollars?
3. To what whole number is $\frac{12}{3}$ equal?
4. To what mixed number is $\frac{15}{4}$ equal?
5. Reduce $\frac{22}{5}$ to a whole or mixed number.
6. Reduce $\frac{38}{7}$ to a whole or mixed number.

SLATE EXERCISES.

1. Reduce $\frac{25}{3}$ to a whole or mixed number.

ANALYSIS.—Since in 3 thirds there is a unit, or one, in 25 thirds there are as many units as 3 is contained times in 25. Dividing the numerator 25 by the denominator 3, the quotient is 8 and 1 over, or $8\frac{1}{3}$. Therefore, $\frac{25}{3} = 8\frac{1}{3}$.

OPERATION.

$3\overline{)25}$

Ans. $8\frac{1}{3}$

24. How reduce an *improper* fraction to a *whole* or *mixed* number?

Divide the numerator by the denominator, and the quotient will be the number required.

Reduce the following to whole or mixed numbers:

2. $\frac{38}{2}$.	6. $\frac{96}{8}$.	10. $\frac{200}{25}$.	14. $\frac{612}{81}$.
3. $\frac{44}{3}$.	7. $\frac{100}{7}$.	11. $\frac{250}{46}$.	15. $\frac{764}{96}$.
4. $\frac{65}{4}$.	8. $\frac{121}{10}$.	12. $\frac{368}{64}$.	16. $\frac{1000}{125}$.
5. $\frac{81}{5}$.	9. $\frac{180}{12}$.	13. $\frac{500}{75}$.	17. $\frac{3000}{250}$.

18. How many sheets of paper shall I require, to give 5 pupils half a sheet apiece?

19. A man meeting 15 beggars, gave each a quarter of a dollar: how many dollars did he give to all?

CASE V.

To reduce a Whole or Mixed Number to an *Improper Fraction.*

1. How many halves in 2 pears?

ANALYSIS.—In 1 pear there are 2 halves; therefore, in 2 pears there are 2 times 2 halves, which are 4 halves.

2. How many halves in 3 whole ones? In 4? In 5?
3. In 5 how many thirds? In 6? In 7? In 10?
4. How many fourths in 5? In 7? In 11? In 12?
5. How many fifths in 8? In 9? In 12?
6. How many tenths in 7? In 9? In 10?
7. How many thirds in $4\frac{2}{3}$?

ANALYSIS.—Since in 1 there are 3 thirds, in 4 there must be 3 times 4, or 12 thirds, and 2 thirds are 14 thirds. Therefore, in $4\frac{2}{3}$ there are $\frac{14}{3}$.

8. How many fourths in $5\frac{1}{4}$? In $6\frac{3}{4}$?
9. How many sevenths in $5\frac{4}{7}$? In $6\frac{3}{7}$? In $8\frac{6}{7}$?
10. Reduce $8\frac{2}{3}$ to an improper fraction.
11. Reduce $10\frac{4}{5}$ to an improper fraction.
12. Reduce $12\frac{3}{7}$ to an improper fraction.

SLATE EXERCISES.

1. Reduce 17 to fifths.

ANALYSIS.—Since there are 5 fifths in a unit, there must be 5 times as many *fifths* in a number as there are units in that number; and 5 times 17 are 85. Therefore, $17=\frac{85}{5}$.

OPERATION.
$17 \times 5 = 85.$
Ans. $\frac{85}{5}$.

2. Reduce $13\frac{4}{5}$ to an improper fraction; that is, to fifths.

ANALYSIS.—Reasoning as before, in 13 units there are 5 times 13, or 65 fifths, and 4 fifths are 69 fifths. We therefore multiply the whole number 13, by the denominator 5, and adding the 4 fifths to the product, we have $\frac{69}{5}$.

$$\begin{array}{r} 13\frac{4}{5} \\ 5 \\ \hline 69 \end{array}$$

Ans. $\frac{69}{5}$.

25. How reduce a *whole* or *mixed* number to an *improper* fraction?

Multiply the whole number by the given denominator; to the product add the numerator, and place the sum over the denominator.

NOTE.—A *whole* number may be reduced to an improper fraction, by making 1 its denominator. Thus, $3=\frac{3}{1}$; for *multiplying* or *dividing* a number by 1, does not *alter* its value.

3. Reduce 25 to thirds.
4. Reduce 43 to fifths.
5. Reduce 31 to fourths.
6. Reduce 65 to sevenths.

Reduce the following to improper fractions:

7. $11\frac{2}{5}$.	12. $56\frac{4}{7}$.	17. $100\frac{3}{20}$.	22. $1000\frac{3}{7}$.
8. $14\frac{2}{3}$.	13. $68\frac{3}{8}$.	18. $117\frac{2}{25}$.	23. $1064\frac{7}{8}$.
9. $17\frac{3}{4}$.	14. $89\frac{3}{10}$.	19. $245\frac{3}{25}$.	24. $2207\frac{3}{4}$.
10. $31\frac{1}{5}$.	15. $73\frac{7}{12}$.	20. $430\frac{1}{50}$.	25. $3046\frac{3}{5}$.
11. $44\frac{1}{6}$.	16. $97\frac{2}{15}$.	21. $507\frac{21}{100}$.	26. $5231\frac{7}{10}$.

27. A gentleman having $25\frac{3}{4}$ dollars, divided it equally among a company of beggars, giving each 1 fourth of a dollar: how many were there in the company?

CASE VI.

To reduce a Compound Fraction to a *Simple one.*

1. To what is $\frac{2}{3}$ of $\frac{1}{5}$ equal?

ANALYSIS.—$\frac{1}{3}$ of $\frac{1}{5}$ is equal to $\frac{1}{15}$; for multiplying the denominator divides the fraction. Now, if $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, 2 thirds will be twice as much, and 2 times $\frac{1}{15}$ are $\frac{2}{15}$. In the operation, *we multiply the numerators together for the new numerator, and the denominators together for the new denominator.*

OPERATION.
$\frac{2}{3}$ of $\frac{1}{5}=\frac{2}{15}$.

2. Reduce $\frac{2}{5}$ of $\frac{1}{4}$ to a simple fraction?

SOLUTION.—$\frac{2}{5}\times\frac{1}{4}=\frac{2}{20}$, or $\frac{1}{10}$, *Ans.*

3. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

4. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

Operation.

$$\frac{1}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{1}{5}.$$

Analysis.—We have seen that dividing the numerator and denominator by the same number does not alter the value of the fraction; also that cancelling a factor divides a number by that factor. We therefore cancel the common factors 3 and 4, then multiplying the factors remaining in the numerators together for the new numerator, and those remaining in the denominators for the new denominator, we have $\frac{1}{5}$ for the simple fraction required.

26. How reduce a compound fraction to a simple one?

Cancel the common factors, and place the product of the factors remaining in the numerators over the product of those remaining in the denominators.

Note.—The object of cancelling the common factors is twofold; it *shortens* the operation, and reduces the result to the *lowest terms.*

5. Reduce $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{3}{4}$ to a simple fraction. *Ans.* $\frac{1}{10}$.

Reduce the following to simple fractions:

6. $\frac{2}{3}$ of $\frac{1}{7}$ of $\frac{3}{4}$.	11. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{5}$.	16. $\frac{4}{5}$ of $\frac{2}{7}$ of 11.
7. $\frac{5}{6}$ of $\frac{1}{3}$ of $\frac{7}{8}$.	12. $\frac{1}{2}$ of $\frac{2}{3}$ of $4\frac{1}{2}$.	17. $\frac{3}{4}$ of $\frac{1}{5}$ of $6\frac{1}{4}$.
8. $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{1}{3}$.	13. $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{7}{11}$.	18. $\frac{2}{3}$ of $\frac{5}{6}$ of 9.
9. $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{5}{9}$.	14. $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{7}{9}$.	19. $\frac{3}{4}$ of $\frac{4}{5}$ of $2\frac{1}{2}$.
10. $\frac{5}{9}$ of $\frac{1}{3}$ of $\frac{6}{8}$.	15. $\frac{2}{5}$ of $\frac{15}{16}$ of $\frac{1}{2}$.	20. $\frac{7}{8}$ of $\frac{4}{5}$ of $10\frac{1}{7}$.

CASE VII.

To reduce a Fraction to any *Required* Denominator.

1. Reduce $\frac{3}{4}$ to *twelfths.*

Operation.

$12 \div 4 = 3.$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}, \textit{ Ans.}$$

Analysis.—The given denominator 4 is contained in 12, the required denominator, 3 times; therefore, multiplying both terms of $\frac{3}{4}$ by 3, it becomes $\frac{9}{12}$, and is the fraction required.

27. How reduce a fraction to any *required* denominator?

Multiply both terms of the fraction by such a number as will make the given denominator equal to the required denominator. (P. 95, Q. 20, *n.*)

Reduce the following fractions to the denominators indicated:

2. $\frac{4}{5}$ to 30ths.	5. $\frac{7}{24}$ to 72ds.	8. $\frac{29}{57}$ to 171sts.
3. $\frac{5}{8}$ to 40ths.	6. $\frac{17}{33}$ to 99ths.	9. $\frac{75}{92}$ to 276ths.
4. $\frac{3}{7}$ to 35ths.	7. $\frac{27}{48}$ to 144ths.	10. $\frac{100}{200}$ to 1000ths.

CASE VIII.

To reduce Fractions to a *Common* Denominator.

28. What is a Common Denominator?

A ***Common Denominator*** is one that *belongs equally to two or more fractions;* as, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

1. It is required to reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, to equivalent fractions, having a *common* denominator.

ANALYSIS.—If we multiply *each denominator* by all the other denominators, the several products will be the *same;* for, each is composed of the *same factors,* 2, 3, and 4, the product of which is 24. Again, if we multiply *each numerator* by all the denominators except its own, it follows that the *terms* of each fraction will be multiplied by the *same number;* therefore, the value of the fractions is not altered. (P. 93, Prin. V.)

OPERATION.

$$\tfrac{1}{2}=\frac{1\times3\times4}{2\times3\times4}=\tfrac{12}{24}.$$

$$\tfrac{1}{3}=\frac{1\times2\times4}{3\times2\times4}=\tfrac{8}{24}.$$

$$\tfrac{1}{4}=\frac{1\times2\times3}{4\times2\times3}=\tfrac{6}{24}.$$

29. How reduce fractions to a common denominator?

Multiply the terms of each fraction by all the denominators except its own.

NOTES.—1. *Mixed* numbers must be reduced to *improper* fractions, and *compound* fractions to *simple* ones, before applying the rule. (Ex. 12.)

2. It should be observed that the *value* of the given fractions is not *altered* by reducing them to a common denominator. The reason is, that the terms of each fraction are multiplied by the same numbers. (P. 93, Prin. V.)

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, to a common denominator.

Reduce the following to a common denominator:

3. $\frac{2}{3}$ and $\frac{3}{4}$.
4. $\frac{2}{5}$ and $\frac{1}{3}$.
5. $\frac{3}{4}$ and $\frac{3}{7}$.
6. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$.
7. $\frac{1}{7}$, $\frac{1}{2}$, $\frac{4}{5}$.
8. $\frac{2}{9}$, $\frac{2}{3}$, $\frac{7}{8}$.
9. $\frac{5}{11}$, $\frac{8}{9}$, $\frac{3}{7}$.
10. $\frac{21}{43}$, $\frac{15}{20}$, $\frac{19}{74}$.
11. $\frac{19}{37}$, $\frac{50}{100}$, $\frac{7}{10}$.

12. Find a common denominator of $\frac{1}{2}$ of $\frac{3}{4}$, $3\frac{2}{3}$ and 5.

ANALYSIS.—$\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$; $3\frac{2}{3}=\frac{11}{3}$, and $5=\frac{5}{1}$. Now, $\frac{3}{8}$, $\frac{11}{3}$, and $\frac{5}{1}$ by the rule, become $\frac{9}{24}$, $\frac{88}{24}$, $\frac{120}{24}$.

13. Reduce $2\frac{1}{2}$, $\frac{3}{4}$ of $\frac{4}{5}$, and 3 to a common denominator.

14. Reduce $\frac{3}{7}$ of $\frac{5}{9}$, 7, and $5\frac{1}{4}$ to a common denominator.

CASE IX.

To reduce Fractions to the *Least Common* Denominator.

80. What is the Least Common Denominator of two or more fractions.

The ***Least Common Denominator*** of two or more fractions is the *least common multiple* of their denominators. (P. 86, Q. 17.)

1. What is the least common denominator of $\frac{2}{3}$, $\frac{3}{10}$, and $\frac{4}{15}$?

ANALYSIS.—The solution of this and similar Examples requires two steps: 1st. To find the *least com. multiple* of the denominators for the required denominator. 2d. To *reduce* the given fractions to *this denominator*.

3) 3, 10, 15
5) 1, 10, 5
1, 2, 1.

$3 \times 5 \times 2 = 30$, *L. C. M.*

The least com. multiple of 3, 10, and 15 is 30. (P. 87.) To reduce $\frac{2}{3}$, $\frac{3}{10}$, and $\frac{4}{15}$ to thirtieths, we multiply both terms of the given fractions by such a number as will reduce them to thirtieths. Now multiplying both terms of $\frac{2}{3}$ by 10, the fraction becomes $\frac{20}{30}$; multiplying both terms of $\frac{3}{10}$ by 3, the fraction becomes $\frac{9}{30}$; and multiplying both terms of $\frac{4}{15}$ by 2, we have $\frac{8}{30}$. (P. 101, Q. 27.) Therefore, the required fractions are $\frac{20}{30}$, $\frac{9}{30}$, and $\frac{8}{30}$.

$$\left.\begin{array}{l} \frac{2\times 10}{3\times 10}=\frac{20}{30}. \\[1ex] \frac{3\times 3}{10\times 3}=\frac{9}{30}. \\[1ex] \frac{4\times 2}{15\times 2}=\frac{8}{30}. \end{array}\right\} \textit{Ans.}$$

31. How reduce fractions to the least common denominator?

Find the least common multiple of all the denominators; then multiply both terms of each fraction by such a number as will reduce it to this denominator. (P. 101, Q. 27.)

NOTE.—*Mixed* numbers must be reduced to *improper* fractions, *compound* fractions to *simple* ones, and all fractions to their *lowest terms*, before applying the rule. (Ex. 12.)

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ to the least com. denominator.

SOLUTION.—The least com. multiple of 2, 3, and 4 is 12. Now $\frac{1}{2}=\frac{6}{12}$; $\frac{1}{3}=\frac{4}{12}$; and $\frac{3}{4}=\frac{9}{12}$. *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{9}{12}$.

Reduce the following to the least com. denominator:

3. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$.	6. $\frac{3}{7}$, $\frac{2}{5}$, $\frac{7}{10}$.	9. $\frac{4}{20}$, $\frac{6}{36}$, $\frac{13}{39}$.
4. $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{10}$.	7. $\frac{7}{18}$, $\frac{5}{9}$, $\frac{1}{12}$.	10. $\frac{3}{14}$, $\frac{2}{3}$, $\frac{3}{7}$, $\frac{1}{6}$.
5. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{1}{6}$.	8. $\frac{8}{5}$, $\frac{6}{7}$, $\frac{2}{3}$.	11. $\frac{8}{21}$, $\frac{7}{18}$, $\frac{5}{33}$, $\frac{7}{9}$.

12. Find the least com. denominator of $2\frac{1}{5}$, $\frac{1}{2}$ of $\frac{2}{10}$, $\frac{3}{4}$ and 5.

ANALYSIS.—$2\frac{1}{5}=\frac{11}{5}$; $\frac{1}{2}$ of $\frac{2}{10}=\frac{2}{20}=\frac{1}{10}$; $\frac{3}{4}=\frac{3}{4}$, and $5=\frac{5}{1}$. The least com. multiple of 5, 10, 4, and 1, is 20. Now $\frac{11}{5}=\frac{44}{20}$; $\frac{1}{10}=\frac{2}{20}$; $\frac{3}{4}=\frac{15}{20}$; and $\frac{5}{1}=\frac{100}{20}$. *Ans.* $\frac{44}{20}$, $\frac{2}{20}$, $\frac{15}{20}$, $\frac{100}{20}$.

13. What is the least com. denominator of $\frac{2}{3}$, $5\frac{1}{4}$, and $\frac{5}{8}$?

14. What is the least com. denominator of $\frac{2}{3}$ of $\frac{3}{4}$, $4\frac{1}{2}$, $\frac{5}{8}$, and 4?

ADDITION OF FRACTIONS.

To add Fractions which have a *Common* Denominator.

REMARK.—When two or more fractions express *parts* of the *same kind* of unit, and have a *common denominator*, their *numerators* are *like numbers*; hence, they may be *added*, *subtracted*, and *divided* as whole numbers.

MENTAL EXERCISES.

1. What is the sum of $\frac{3}{5}$ dollar, $\frac{2}{5}$ dollar, and $\frac{4}{5}$ dollar?

ANALYSIS.—3 fifths dollar and 2 fifths are 5 fifths, and 4 are 9 fifths, which are equal to $1\frac{4}{5}$ dollar.

2. What is the sum of $\frac{3}{4}$, $\frac{2}{4}$, and $\frac{1}{4}$?
3. What is the sum of $\frac{5}{6}$, $\frac{1}{6}$, $\frac{4}{6}$, and $\frac{2}{6}$?
4. What is the sum of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{4}{7}$, and $\frac{5}{7}$?
5. What is the sum of $\frac{3}{8}$, $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$?
6. What is the sum of $\frac{3}{15}$, $\frac{4}{15}$, $\frac{7}{15}$, and $\frac{8}{15}$?

SLATE EXERCISES.

32. When fractions have a *common denominator*, what is true of their numerators?

Their *numerators* express *like parts* of a unit, and therefore are *like numbers*.

1. What is the sum of $\frac{13}{20}$, $\frac{16}{20}$, and $\frac{14}{20}$?

ANALYSIS.—As these fractions have a common denominator, their numerators are *like numbers*. Hence, they may be added as whole numbers. (P. 25. Q. 9.) Thus, the sum of 13 twentieths + 16 twentieths + 14 twentieths = $\frac{43}{20}$, or $2\frac{3}{20}$, *Ans.*

OPERATION.

$$\frac{13}{20}+\frac{16}{20}+\frac{14}{20}=\frac{43}{20}, \text{ or } 2\frac{3}{20}.$$

33. How add fractions which have a *common denominator?*

Add the numerators, and place the sum over the common denominator.

NOTE.—The answers should be reduced to the *lowest* terms, and *improper* fractions to *whole* or *mixed* numbers.

2. A man sold $\frac{5}{8}$ of an acre of land to one customer, $\frac{7}{8}$ to another, $\frac{3}{8}$ to another, and $\frac{6}{8}$ to another: how much did he sell to all?

3. What is the sum of $\frac{5}{16}$ pound, $\frac{7}{16}$ pound, $\frac{11}{16}$ pound, and $\frac{9}{16}$ pound?

4. What is the sum of $\frac{8}{20}$, $\frac{13}{20}$, $\frac{15}{20}$, $\frac{17}{20}$?

5. What is the sum of $\frac{11}{25}$, $\frac{7}{25}$, $\frac{14}{25}$, $\frac{21}{25}$?

6. What is the sum of $\frac{9}{30}$, $\frac{12}{30}$, $\frac{17}{30}$, $\frac{21}{30}$?

To add Fractions which have *Different* Denominators.

1. What is the sum of $\frac{1}{2}$ dollar, $\frac{3}{4}$ dollar, and $\frac{5}{6}$ dollar?

ANALYSIS.—Since these fractions have different denominators, their *numerators* denote *unlike* parts of a unit; consequently, they cannot be added, any more than units of different orders. We therefore reduce them to a common denominator, which is 48, and add the numerators, as above.

OPERATION.

$2 \times 4 \times 6 = 48$, *Com. D.*
$1 \times 4 \times 6 = 24$, *1st N.*
$3 \times 2 \times 6 = 36$, *2d N.*
$5 \times 2 \times 4 = 40$, *3d N.*
$\frac{24}{48} + \frac{36}{48} + \frac{40}{48} = \frac{100}{48}$, or $2\frac{1}{12}$.

Or, we may reduce the fractions to the *least common* denominator, which is 12, and then add the numerators. Thus, $\frac{1}{2} = \frac{6}{12}$; $\frac{3}{4} = \frac{9}{12}$; and $\frac{5}{6} = \frac{10}{12}$: now $\frac{6}{12} + \frac{9}{12} + \frac{10}{12} = \frac{25}{12}$, or $2\frac{1}{12}$, the same as before.

34. What then is the general rule for adding fractions?

Reduce them to a common denominator, and place the sum of the numerators over it.

Or, reduce them to the least common denominator, and over this, place the sum of the numerators.

NOTE.—The *integral* and *fractional* parts of *mixed* numbers should be added separately, and the results be united. (Ex. 12.)

Or, *mixed* numbers may be reduced to *improper* fractions, and *compound* fractions to *simple* ones, and then be added. (Ex. 18.)

2. Henry paid $\frac{1}{2}$ dollar for an arithmetic, $\frac{1}{6}$ dollar for a slate, and $\frac{3}{8}$ dollar for a geography: what did he pay for all? *Ans.* $1\frac{1}{24}$ dol.

3. What is the sum of $\frac{3}{5}$ pound, $\frac{5}{8}$ pound, and $\frac{7}{10}$ pound?

4. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
5. Add $\frac{2}{5}$, $\frac{4}{7}$, and $\frac{1}{2}$.
6. Add $\frac{5}{11}$, $\frac{1}{3}$, and $\frac{15}{33}$.
7. Add $\frac{7}{15}$, $\frac{19}{20}$, and $\frac{11}{12}$.
8. Add $\frac{3}{10}$, $\frac{7}{8}$, and $\frac{4}{5}$.
9. Add $\frac{9}{16}$, $\frac{17}{20}$, and $\frac{11}{40}$.
10. Add $\frac{13}{25}$, $\frac{7}{10}$, and $\frac{45}{50}$.
11. Add $\frac{14}{36}$, $\frac{19}{24}$, and $\frac{45}{48}$.

12. What is the sum of $10\frac{1}{4}$ pounds, $17\frac{3}{5}$, and $23\frac{5}{8}$ pounds?

ANALYSIS.—Reducing the fractional parts to the least common denominator, which is 40, we add the fractions and integers separately.

The sum of the fractions is $\frac{59}{40}=1\frac{19}{40}$. Adding the 1 to the whole number, the sum is $51\frac{19}{40}$, *Ans.*

$$
\begin{aligned}
10\tfrac{1}{4} &= 10\tfrac{10}{40}\\
17\tfrac{3}{5} &= 17\tfrac{24}{40}\\
23\tfrac{5}{8} &= 23\tfrac{25}{40}\\
\hline
\textit{Ans. } & 51\tfrac{19}{40}
\end{aligned}
$$

13. How many pounds of tea are there in 2 chests, containing $45\frac{1}{2}$ and $56\frac{1}{3}$ pounds respectively?

14. How much cloth in 3 pieces, containing $12\frac{1}{2}$, $17\frac{3}{4}$, and $21\frac{5}{6}$ yards?

15. If a man walks $20\frac{2}{3}$ miles in one day, $25\frac{1}{2}$ the next, and $31\frac{3}{4}$ the next, how far will he travel in all?

16. If a housekeeper buys $3\frac{1}{2}$ dollars worth of sugar, $5\frac{3}{4}$ dollars worth of coffee, and $15\frac{5}{8}$ dollars worth of flour, what is the amount of her bill?

17. Three men buying a sail-boat, put in $27\frac{1}{4}$, $23\frac{5}{8}$, and $20\frac{5}{6}$ dollars respectively: what was the cost of the boat?

18. What is the sum of $\frac{1}{2}$ of $\frac{2}{5}$, $\frac{1}{4}$ of $\frac{4}{20}$, and $\frac{1}{2}$ of $2\frac{1}{2}$?

SOLUTION.—Reducing the compound fractions to simple ones, we have $\frac{1}{2}$ of $\frac{2}{5}=\frac{1}{5}$, $\frac{1}{4}$ of $\frac{4}{20}=\frac{1}{20}$, and $\frac{1}{2}$ of $2\frac{1}{2}=\frac{5}{4}$. Reducing these fractions to the least common denominator, 20, they become $\frac{4}{20}$, $\frac{1}{20}$, and $\frac{25}{20}$; and the sum $\frac{4}{20}+\frac{1}{20}+\frac{25}{20}=\frac{30}{20}$, or $1\frac{1}{2}$, *Ans.*

19. What is the sum of $\frac{2}{3}$ of $\frac{1}{4}$, $\frac{3}{4}$ of $\frac{12}{20}$, and $5\frac{1}{4}$?
20. Add $\frac{3}{5}$ of 45, $\frac{7}{15}$ of $\frac{5}{21}$, and $\frac{5}{10}$ of $3\frac{1}{3}$.
21. Add $22\frac{1}{2}$, $\frac{3}{4}$ of $\frac{2}{3}$, and $\frac{7}{9}$ of $\frac{3}{14}$.
22. What is the sum of $\frac{5}{8}$ of 4, $28\frac{3}{4}$, and $15\frac{2}{3}$?

SUBTRACTION OF FRACTIONS.

To Subtract Fractions which have a *Common* Denominator.

1. If Frank has $\frac{4}{5}$ of a pound of maple sugar, and gives away $\frac{3}{5}$ of it, how much will he have left?

ANALYSIS.—3 fifths from 4 fifths leaves 1 fifth. Therefore, he will have 1 fifth of a pound left.

2. From $\frac{7}{8}$ yard, take $\frac{3}{8}$ yard?

3. What is the difference between $\frac{9}{10}$ of a dime and $\frac{5}{10}$ of a dime?

4. What is the difference between $\frac{3}{11}$ and $\frac{8}{11}$?

5. What is the difference between $\frac{2}{7}$ of a week and $\frac{5}{7}$ of a week?

6. What is the difference between $\frac{5}{16}$ and $\frac{11}{16}$?

SLATE EXERCISES.

1. What is the difference between $\frac{4}{12}$ of a foot and $\frac{11}{12}$ of a foot?

ANALYSIS.—Since these fractions have a common denominator, their numerators are *like numbers*, and may be *subtracted* as whole numbers. (P. 104, Rem.) $\frac{11}{12}$ minus $\frac{4}{12}$ equal $\frac{7}{12}$ foot, *Ans.*

OPERATION.
$\frac{11}{12}-\frac{4}{12}=\frac{7}{12}$ ft.

35. How subtract fractions which have a *common* denominator?

Take the less numerator from the greater, and place the difference over the common denominator.

2. From $\frac{17}{24}$ of a day, subtract $\frac{11}{24}$ of a day. *Ans.* $\frac{1}{4}$ d.

3. From $\frac{31}{40}$ of a ton, subtract $\frac{19}{40}$ of a ton.

4. From $\frac{48}{63}$ of a bushel, subtract $\frac{31}{63}$ of a bushel.

5. From $\frac{110}{263}$, subtract $\frac{50}{263}$.

6. From $\frac{567}{1219}$, subtract $\frac{342}{1219}$.

7. What is the difference between $\frac{746}{1500}$ and $\frac{835}{1500}$?

8. What is the difference between $\frac{1345}{1873}$ and $\frac{1256}{1873}$?

To Subtract Fractions which have *Different* Denominators.

1. It is required to find the difference between $\frac{3}{8}$ and $\frac{10}{12}$.

ANALYSIS.—Since these fractions have *not a common* denominator, their *numerators* are *unlike numbers;* consequently one cannot be taken directly from the other. Hence, we reduce them to a *common* denominator, and subtract as above.

OPERATION.

$8 \times 12 = 96$, *C. D.*

$\frac{3}{8} = \frac{36}{96}$; and $\frac{10}{12} = \frac{80}{96}$.

$\frac{80}{96} - \frac{36}{96} = \frac{44}{96}$, or $\frac{11}{24}$.

36. What then is the general Rule for subtracting fractions?

Reduce them to a common denominator, and over it place the difference of the numerators.

NOTES.—1. *Whole* and *mixed* numbers should be reduced to *improper* fractions, and *compound* fractions to *simple* ones; then proceed as above. (Ex. 15, 23.)

2. If both are mixed numbers, it is sometimes more expeditious to reduce the fractions to a common denominator; then subtract the fractional and integral parts separately. (Ex. 17.)

3. The operation may often be shortened by reducing the fractions to the *least common denominator*.

2. Bought a cargo of corn, at $\frac{5}{8}$ of a dollar a bushel, and sold it at $\frac{3}{4}$ of a dollar: what was the gain per bushel?

3. A man owning $\frac{15}{16}$ of a ship, sold $\frac{5}{8}$ of her: what part had he left?

4. From $\frac{7}{8}$, take $\frac{4}{5}$.
5. From $\frac{11}{12}$, take $\frac{5}{8}$.
6. From $\frac{15}{24}$, take $\frac{17}{38}$.
7. From $\frac{25}{48}$, take $\frac{5}{36}$.
8. From $\frac{27}{30}$, take $\frac{2}{3}$.
9. From $\frac{51}{72}$, take $\frac{3}{8}$.
10. From $\frac{45}{80}$, take $\frac{21}{60}$.
11. From $\frac{76}{100}$, take $\frac{13}{75}$.

12. What is the difference between $\frac{125}{250}$ and $\frac{63}{135}$?
13. What is the difference between $\frac{200}{600}$ and $\frac{94}{450}$?
14. What is the difference between $\frac{350}{600}$ and $\frac{430}{800}$?

15. Subtract $14\frac{5}{7}$ hogsheads from 36 hogsheads?

ANALYSIS.—Reducing 36 and $14\frac{5}{7}$ to improper fractions, we have $\frac{252}{7}$ and $\frac{103}{7}$. Now 252 minus 103 equals $\frac{149}{7}$; and $\frac{149}{7}$ equals $21\frac{2}{7}$. *Ans.* $21\frac{2}{7}$ hogsheads.

Or, borrowing 1, which equals $\frac{7}{7}$, we have $\frac{7}{7}$ minus $\frac{5}{7}$, equal to $\frac{2}{7}$. Then 1 to carry to 14 makes 15; and 36 minus 15=21. *Ans.* $21\frac{2}{7}$.

1ST OPERATION.

$36=\frac{252}{7}$.

$14\frac{5}{7}=\frac{103}{7}$.

$252-103=\frac{149}{7}$, or $21\frac{2}{7}$.

2D OPERATION.

$$\begin{array}{r} 36 \\ \underline{14\frac{5}{7}} \end{array}$$

Ans. $21\frac{2}{7}$ h.

16. A farmer having 85 bushels of wheat, sold $63\frac{5}{8}$ bushels: how much had he left?

17. What is the difference between $21\frac{2}{3}$ tons and $16\frac{4}{5}$ tons?

ANALYSIS.—Reducing the fractions to the common denominator 15, the minuend $21\frac{2}{3}$ $=21\frac{10}{15}$; and the subtrahend $16\frac{4}{5}=16\frac{12}{15}$. Now $\frac{12}{15}$ is larger than $\frac{10}{15}$, the fraction above it; hence we borrow 1, or $\frac{15}{15}$, and add it to $\frac{10}{15}$, making $\frac{25}{15}$; and $\frac{25}{15}-\frac{12}{15}=\frac{13}{15}$; carrying 1 to 16 makes 17, and $21-17=4$. *Ans.* $4\frac{13}{15}$ tons.

OPERATION.

$21\frac{2}{3}=21\frac{10}{15}$.

$16\frac{4}{5}=16\frac{12}{15}$.

Ans. $4\frac{13}{15}$ tons.

18. From a cask of molasses, containing $56\frac{3}{8}$ gallons, $20\frac{1}{2}$ gallons were drawn: how many remained?

19. Take $18\frac{3}{4}$ from $37\frac{1}{2}$.

20. Take $31\frac{1}{4}$ from $66\frac{2}{3}$.

21. Take $62\frac{1}{2}$ from $83\frac{1}{4}$.

22. Take $106\frac{1}{4}$ from $135\frac{3}{8}$.

23. Required the difference between $\frac{1}{5}$ of $\frac{2}{3}$ of 5, and $\frac{1}{4}$ of $\frac{4}{9}$ of $3\frac{3}{5}$.

SOLUTION.—Reducing the compound fractions to simple ones, cancelling, etc., the minuend becomes $\frac{2}{3}$, and the subtrahend $\frac{2}{5}$. Again, reducing $\frac{2}{3}$ and $\frac{2}{5}$ to the common denominator 15, we obtain $\frac{10}{15}$, and $\frac{6}{15}$; and $\frac{10}{15}-\frac{6}{15}=\frac{4}{15}$, *Ans.*

24. From $\frac{2}{3}$ of $\frac{3}{4}$, take $\frac{1}{8}$ of $\frac{4}{5}$.

25. From $\frac{5}{8}$ of $\frac{4}{10}$, take $\frac{5}{12}$ of $\frac{3}{15}$.

MULTIPLICATION OF FRACTIONS.

CASE I.

To Multiply a Fraction by a *Whole* Number.

1. At $\frac{1}{2}$ cent apiece, what will 3 plums cost?

ANALYSIS.—Since 1 plum costs $\frac{1}{2}$ cent, 3 plums will cost 3 times as much; and 3 times 1 half are 3 halves, equal to $1\frac{1}{2}$ cent. Therefore, 3 plums will cost $1\frac{1}{2}$ cent.

2. At $\frac{2}{3}$ of a dollar a pound, what will 3 pounds of tea come to?

3. If a lad earn $\frac{3}{4}$ of a dollar per day, how much will he earn in 5 days?

4. What cost 6 bushels of apples, at $\frac{5}{8}$ of a dollar a bushel?

5. What is the product of 5 times $\frac{6}{11}$? Of 7 times $\frac{8}{9}$?

6. What cost 12 photographs, at $\frac{3}{8}$ dollar apiece?

7. What cost 11 rabbits, at $\frac{5}{7}$ dollar apiece?

8. What cost 5 oranges, at $6\frac{1}{4}$ cents each?

ANALYSIS.—If 1 orange costs $6\frac{1}{4}$ cents, 5 will cost 5 times $6\frac{1}{4}$ cents. Now, 5 times 6 cents are 30 cents, and 5 times $\frac{1}{4}$ are $\frac{5}{4}$, equal to $1\frac{1}{4}$, which added to 30 make $31\frac{1}{4}$ cents. Therefore, etc.

9. At $3\frac{1}{2}$ dimes each, what will 9 melons cost?

10. What cost 12 gold pens, at $4\frac{2}{3}$ dollars apiece?

SLATE EXERCISES.

1. At $\frac{5}{6}$ dollar a box, what will 3 boxes of starch cost?

ANALYSIS.—Since 1 box costs $\frac{5}{6}$ dollar, 3 boxes will cost 3 times $\frac{5}{6}$ dol.; and 3 times $\frac{5}{6}=\frac{15}{6}$, or $2\frac{1}{2}$ dollars, the cost required.

1ST OPERATION.

$\frac{5}{6}\times 3=\frac{15}{6}$, or $2\frac{1}{2}$ d.

Or, if we divide the denominator by 3, the result will be the same; for, dividing the denominator multiplies the fraction.

2D OPERATION.

$\frac{5}{6}\div 3=\frac{5}{2}$, or $2\frac{1}{2}$ d.

37. How multiply a *fraction* by a *whole* number?

Multiply the numerator by the whole number.

Or, divide the denominator by it. (P. 93, Prin. IV.)

NOTES.—1. The *second* method is preferable, when the denominator can be divided by the whole number without a remainder.

2. If the multiplicand is a *mixed* number, multiply the *fractional* and *integral* parts separately, and *unite* the products.

Or, reduce the mixed number to an improper fraction; then apply the rule. (Ex. 15.)

2. What will 17 pounds of honey cost, at $\frac{3}{8}$ of a dollar a pound?

3. What cost 25 bushels of potatoes, at $\frac{5}{12}$ of a dollar a bushel?

4. At $4\frac{9}{16}$ dollars apiece, what will 33 straw hats come to?

5. Multiply $\frac{11}{13}$ by 15.
6. Multiply $\frac{15}{16}$ by 17.
7. Multiply $\frac{19}{20}$ by 9.
8. Multiply $\frac{20}{31}$ by 11.
9. Multiply $\frac{13}{35}$ by 18.
10. Multiply $\frac{21}{44}$ by 26.
11. Multiply $\frac{35}{56}$ by 42.
12. Multiply $\frac{61}{83}$ by 50.
13. Multiply $\frac{75}{100}$ by 83.
14. Multiply $\frac{100}{256}$ by 110.

15. What will 5 hundred weight of sugar cost, at $6\frac{3}{4}$ dollars per hundred?

ANALYSIS.—Multiplying the fraction and integer separately by the whole number, we have $\frac{3}{4}\times 5=\frac{15}{4}$, or $3\frac{3}{4}$; and $6\times 5=30$. Now $30+3\frac{3}{4}=33\frac{3}{4}$ dols. Therefore, etc.

Or, the mixed number $6\frac{3}{4}=\frac{27}{4}$, and $\frac{27}{4}\times 5=\frac{135}{4}=33\frac{3}{4}$ dollars, the cost required.

OPERATION.

$$\begin{array}{l} 6\frac{3}{4} \text{ dols.} \\ \underline{5} \\ 33\frac{3}{4} \text{ dols.} \end{array}$$

16. What cost 23 yards of muslin, at $12\frac{1}{2}$ cents a yard?

17. What cost 45 yearlings, at $18\frac{3}{4}$ dollars apiece?

18. Multiply $31\frac{1}{4}$ by 25.
19. Multiply $37\frac{1}{2}$ by 42.
20. Multiply $40\frac{5}{8}$ by 61.
21. Multiply $62\frac{1}{2}$ by 57.
22. Multiply $66\frac{2}{3}$ by 75.
23. Multiply $87\frac{9}{12}$ by 100.

24. What cost 24 bureaus, at $27\frac{1}{4}$ dollars apiece?

25. What cost 12 melodeons, at $62\frac{1}{2}$ dollars apiece?

26. What cost 31 sofas, at $71\frac{3}{4}$ dollars apiece?

CASE II.

To Multiply a Whole Number by a Fraction.

1. What will $\frac{1}{2}$ yard of edging cost, at 10 cents a yard?

ANALYSIS.—If 2 halves, or a whole yard, cost 10 cents, 1 half yard will cost 1 half of 10 cts.; which is 5 cts. Therefore, etc.

2. If a dozen eggs cost 16 cts., what will $\frac{1}{2}$ dozen cost?
3. If a melon is worth 12 cents, what is $\frac{1}{3}$ of it worth?
4. If a pie is worth 20 cents, what is $\frac{1}{4}$ of it worth?
5. What cost $\frac{2}{3}$ pound of grapes, at 14 cents a pound?

ANALYSIS.—Since 1 pound is worth 14 cents, $\frac{2}{3}$ of a pound are worth $\frac{2}{3}$ of 14 cents. But 1 third of 14 cents is $4\frac{2}{3}$ cents, and 2 thirds are 2 times $4\frac{2}{3}$, or $9\frac{1}{3}$ cents. Therefore, etc.

6. If a cake costs 80 cents, what will $\frac{3}{4}$ of it cost?

SLATE EXERCISES.

38. What is meant by multiplying by a fraction?

Multiplying by a Fraction is taking a *certain part* of the multiplicand as many times as there are *like parts* of a *unit* in the multiplier.

39. How find a fractional part of a number?

Divide the number into as many equal parts as there are *units* in the denominator, and then take as many of these parts as there are *units* in the numerator. That is,

To multiply a number by $\frac{1}{2}$, *divide* it by 2.

To multiply a number by $\frac{1}{3}$, *divide* it by 3.

To multiply a number by $\frac{3}{4}$, *divide* it by 4 for $\frac{1}{4}$, and multiply this quotient by 3 for $\frac{3}{4}$, etc.

REMARKS.—1. Multiplying a *whole number* by a *fraction* is the same as *taking* a corresponding *fractional part* of the number.

2. When the *multiplier* is 1, the *product* is *equal* to the *multiplicand;* when the multiplier is *greater* than 1, the product is *greater* than the multiplicand; when the multiplier is *less* than 1, the product is *less* than the multiplicand.

1. What will $\frac{2}{3}$ of a gallon of cider cost, at 38 cents a gallon?

ANALYSIS.—Since 1 gallon costs 38 cents, $\frac{2}{3}$ of a gallon must cost $\frac{2}{3}$ times 38, or $\frac{2}{3}$ of 38 cts. Now $\frac{1}{3}$ of 38 cts. is $12\frac{2}{3}$ cts., and 2 thirds are 2 times $12\frac{2}{3}$ cts. Multiplying $12\frac{2}{3}$ by 2, we have 2 times $\frac{2}{3}=\frac{4}{3}$, or $1\frac{1}{3}$. 2 times 12 are 24, and $1\frac{1}{3}$ make $25\frac{1}{3}$ cts., the cost required.

Or, thus: $\frac{2}{3}$ of a gallon will cost $\frac{1}{3}$ of 2 times the cost of 1 gallon. Now 2 times 38 cts. are 76 cts., and $\frac{1}{3}$ of 76 cts. equals 76+3, or $25\frac{1}{3}$ cts., the same as before.

1ST OPERATION.

$$\begin{array}{r} 3\,)\,38 \\ \hline 12\frac{2}{3} \\ 2 \\ \hline \end{array}$$

Ans. $25\frac{1}{3}$ cts.

2D OPERATION.

$38 \times 2 = 76.$

$76 \div 3 = 25\frac{1}{3}$ cts.

40. How multiply a *whole* number by a *fraction?*

Divide the whole number by the denominator of the fraction, and multiply by the numerator.

Or, multiply the whole number by the numerator of the fraction, and divide by the denominator.

NOTES.—1. The *fraction* may be taken for the *multiplicand*, and the *whole number* for the *multiplier*, at pleasure, without affecting the result. (P. 47, Rem.)

2. *Mixed numbers*, when multipliers, may be reduced to *improper fractions;* then proceed according to the rule.

Or, multiply by the *fractional* and *integral* parts separately, and unite the results.

2. If a bushel of barley is worth 75 cts., what is $\frac{3}{5}$ of a bushel worth?

3. If an acre of land is worth \$100, what is $\frac{5}{8}$ of an acre worth?

4. Multiply 45 by $\frac{3}{4}$.

5. Multiply 61 by $\frac{4}{5}$.

6. Multiply 78 by $\frac{2}{7}$.

7. Multiply 87 by $\frac{3}{8}$.

8. Multiply 110 by $\frac{3}{11}$.

9. Multiply 238 by $\frac{5}{16}$.

10. Multiply 378 by $\frac{17}{25}$.

11. Multiply 500 by $\frac{27}{100}$.

12. In 1 year there are 365 days: how many days are there in $\frac{4}{5}$ of a year?

13. What cost $6\frac{4}{5}$ tons of iron, at 42 dollars a ton?

ANALYSIS.—If 1 ton costs 42 dollars, $6\frac{4}{5}$ tons will cost $6\frac{4}{5}$ times 42 dols. We first multiply by the whole number 6, and the product is 252. In multiplying by the fraction $\frac{4}{5}$, we take $\frac{1}{5}$ of the multiplicand, and setting it under the product of the integral part, multiply it by 3; for, $\frac{4}{5}=\frac{1}{5}+\frac{3}{5}$. We now have the partial products of 6, of $\frac{1}{5}$, and $\frac{3}{5}$, and their sum, $285\frac{3}{5}$ dols., is the answer required.

OPERATION.

5)42	dols.	1	ton.
$6\frac{4}{5}$			
252	"	6	"
$8\frac{2}{5}$	"	$\frac{1}{5}$	"
$25\frac{1}{5}$	"	$\frac{3}{5}$	"
$285\frac{3}{5}$	dols.	$6\frac{4}{5}$	tons.

14. What cost $8\frac{3}{4}$ yards of alpaca, at 80 cts. a yard?

15. If a man can walk 45 miles a day, how far can he walk in $10\frac{4}{5}$ days?

16. Multiply 52 by $6\frac{1}{4}$.
17. Multiply 57 by $7\frac{3}{4}$.
18. Multiply 78 by $8\frac{1}{2}$.
19. Multiply 101 by $10\frac{1}{4}$.
20. Multiply 365 by $11\frac{3}{7}$.
21. Multiply 500 by $12\frac{1}{2}$.

CASE III.

To Multiply a Fraction by a Fraction.

1. What will $\frac{4}{10}$ of a gallon of syrup cost, at $\frac{5}{6}$ of a dollar a gallon?

ANALYSIS.—1 tenth of a gallon will cost $\frac{1}{10}$ the price of 1 gal.; and $\frac{1}{10}$ of $\frac{5}{6}$ dol. is $\frac{5}{60}$ dol. (P. 93, Prin. III.)

OPERATION.

$$\frac{\cancel{5}}{\underset{3}{\cancel{6}}}\times\frac{\overset{2}{\cancel{4}}}{\underset{2}{\cancel{10}}}=\frac{1}{3}\text{ dol.}$$

Again, $\frac{4}{10}$ gal. will cost 4 times as much as $\frac{1}{10}$; and 4 times $\frac{5}{60}$ are $\frac{20}{60}$, or $\frac{1}{3}$ dol. In the operation we cancel the common factors, and multiply the numerators together, and then the denominators.

41. How multiply a fraction by a fraction?

Cancel the common factors; then multiply the numerators together for the new numerator, and the denominators for the new denominator.

NOTES.—1. *Compound fractions* are multiplied like *simple* ones; the word *of* being equivalent to the *sign* ×.

2. Reduce mixed numbers to improper fractions, and then multiply them according to the rule. (Ex. 16.)

3. The *object* in cancelling the *common factors* is twofold: it *shortens* the operation, and gives the *answer* in the *lowest* terms.

2. What will $\frac{2}{3}$ of a pound of sugar cost, at $\frac{1}{8}$ of a dollar a pound?

3. What cost $\frac{3}{4}$ of a yard of muslin, at $\frac{2}{10}$ of a dollar a yard?

4. Multiply $\frac{4}{5}$ by $\frac{3}{4}$.
5. Multiply $\frac{5}{9}$ by $\frac{6}{10}$.
6. Multiply $\frac{3}{14}$ by $\frac{7}{9}$.
7. Multiply $\frac{7}{11}$ by $\frac{3}{5}$.
8. Multiply $\frac{12}{21}$ by $\frac{14}{18}$.
9. Multiply $\frac{25}{37}$ by $\frac{19}{75}$.
10. Multiply $\frac{2}{3}$ by $\frac{3}{4} \times \frac{5}{7}$.
11. Multiply $\frac{3}{5}$ by $\frac{7}{9} \times \frac{5}{12}$.
12. Multiply $\frac{2}{7}$ by $\frac{1}{2} \times \frac{7}{15}$.
13. Multiply $\frac{7}{15} \times \frac{5}{14} \times \frac{3}{27}$.
14. Multiply $\frac{1}{2} \times \frac{4}{5} \times \frac{2}{7} \times \frac{5}{8}$.
15. Multiply $\frac{3}{4} \times \frac{7}{9} \times \frac{4}{10} \times \frac{12}{21}$.

16. What cost $8\frac{3}{4}$ yards of calico, at $12\frac{1}{2}$ cents a yard?

SOLUTION—$8\frac{3}{4} = \frac{35}{4}$; $12\frac{1}{2} = \frac{25}{2}$. Now, $\frac{35}{4} \times \frac{25}{2} = \frac{875}{8}$, or $109\frac{3}{8}$ cts.

42. The preceding principles may be summed up in the following

GENERAL RULE.

Reduce whole and mixed numbers to improper fractions; then cancel the common factors, and place the product of the numerators over the product of the denominators.

EXAMPLES FOR PRACTICE.

1. What is the product of $\frac{4}{5} \times \frac{2}{7} \times \frac{5}{6} \times \frac{14}{35}$?
2. Multiply $\frac{3}{4}$ of $\frac{7}{9}$ of $1\frac{1}{2}$ by $\frac{3}{7}$ of $\frac{8}{12}$.
3. Multiply $\frac{2}{5}$ of $\frac{6}{7}$ of $\frac{5}{12}$ by $\frac{2}{3}$ of $4\frac{1}{2}$.
4. Multiply $\frac{5}{7}$ of $6\frac{1}{4}$ by $\frac{3}{5}$ of $\frac{15}{28}$ of 8.
5. Multiply $\frac{2}{5}$ of $\frac{3}{4}$ of 18 by $\frac{3}{8}$ of 25.
6. What is the product of $6\frac{1}{9}$ multiplied by $2\frac{2}{5}$?

7. What cost $10\frac{1}{2}$ pounds of beef, at $15\frac{3}{4}$ cents a pound?

8. At $11\frac{3}{4}$ dollars a barrel, what will $20\frac{1}{2}$ barrels of vinegar come to?

9. Multiply $16\frac{2}{5}$ by $9\frac{3}{4}$.
10. Multiply $31\frac{1}{4}$ by $18\frac{3}{4}$.
11. Multiply $37\frac{1}{2}$ by $16\frac{2}{3}$.
12. Multiply $45\frac{4}{5}$ by $31\frac{1}{4}$.
13. Multiply $66\frac{2}{3}$ by $37\frac{5}{9}$.
14. Multiply $110\frac{7}{9}$ by $60\frac{2}{11}$.

DIVISION OF FRACTIONS.

CASE I.

To Divide a Fraction by a Whole Number.

1. If 2 citrons cost $\frac{4}{10}$ of a dollar, what will 1 citron cost?

ANALYSIS.—If 2 citrons cost $\frac{4}{10}$ of a dollar, 1 will cost 1 half of $\frac{4}{10}$ of a dollar; and 1 half of 4 tenths is $\frac{2}{10}$, or $\frac{1}{5}$ of a dollar.

2. If 3 apples cost $\frac{9}{10}$ of a dime, what will 1 apple cost?

3. If 4 peaches cost $\frac{8}{12}$ of a shilling, what will 1 cost?

4. If 5 yards of calico cost $\frac{10}{12}$ of a dollar, what will 1 yard cost?

5. If 3 doves cost $\frac{1}{2}$ dollar, what will 1 dove cost?

ANALYSIS.—1 dove is $\frac{1}{3}$ of 3 doves; therefore, 1 dove will cost $\frac{1}{3}$ of $\frac{1}{2}$ dollar, and $\frac{1}{3}$ of $\frac{1}{2}$ dollar equals $\frac{1}{6}$ dollar. (P. 63, Q. 12.)

6. If 4 balls cost $\frac{7}{8}$ dollar, what will 1 ball cost?

7. If $\frac{3}{4}$ bushel of oats are equally divided among 5 horses, how many will each horse receive?

8. If $\frac{5}{8}$ of a barrel of apples are divided equally among 7 persons, what part of a barrel will each receive?

9. If $\frac{15}{16}$ of an acre of land are divided into 4 equal lots, how much will there be in each lot?

10. If $\frac{19}{20}$ of a ton of hay are divided into 3 equal loads, how much will there be in each load?

SLATE EXERCISES.

1. If 3 pounds of raisins cost $\frac{6}{7}$ dollar, what will 1 pound cost?

1ST METHOD.—1 pound is $\frac{1}{3}$ of 3 pounds, therefore 1 pound will cost $\frac{1}{3}$ of $\frac{6}{7}$ dol. Dividing the numerator into 3 equal parts, we have $\frac{6}{7}$ dol. $\div 3 = \frac{2}{7}$ dollar. (P. 93, Prin. II.)

2D METHOD.—Since multiplying the denominator divides a fraction, it follows that $\frac{6}{7}$ dol. $\div 3 = \frac{6}{21}$, or $\frac{2}{7}$ dol., the same as before. (P. 93, Prin. III.)

1ST OPERATION.

$\frac{6}{7} \div 3 = \frac{2}{7}$ dol.

2D OPERATION.

$\frac{6}{7} \div 3 = \frac{6}{21}$.

Ans. $\frac{6}{21} = \frac{2}{7}$ dol.

REMARK.—The solution of this and similar examples is an application of the second office of Division. (P. 63, Q. 10.)

43. How divide a *fraction* by a *whole* number?

Divide the numerator by the whole number.

Or, multiply the denominator by it.

NOTES.—1. When the dividend is a *mixed* number, reduce it to an *improper* fraction; then apply the rule. (Ex. 13.)

2. This rule depends upon the principle that a *part* of a unit may be divided into other parts, as well as a *whole* unit.

2. A lad paid $\frac{18}{20}$ dollar for 6 balls: what was that apiece? *Ans.* $\frac{3}{20}$ dol.

3. Divide $\frac{10}{13}$ by 2.
4. Divide $\frac{11}{12}$ by 3.
5. Divide $\frac{15}{19}$ by 6.
6. Divide $\frac{21}{34}$ by 7.
7. Divide $\frac{36}{40}$ by 11.
8. Divide $\frac{78}{43}$ by 31.
9. Divide $\frac{100}{81}$ by 50.
10. Divide $\frac{127}{95}$ by 64.
11. Divide $\frac{250}{112}$ by 85.
12. Divide $\frac{309}{210}$ by 100.

13. A drover paid $18\frac{1}{4}$ dollars for 5 sheep: how much was that per head?

ANALYSIS.—Reducing the mixed number $18\frac{1}{4}$ to an improper fraction, it becomes $\frac{73}{4}$; and $\frac{73}{4} \div 5 = \frac{73}{20}$, or $3\frac{13}{20}$ dols.

OPERATION.

$18\frac{1}{4} = \frac{73}{4}$.

$\frac{73}{4} \div 5 = \frac{73}{20}$, or $3\frac{13}{20}$.

14. If 5 barrels of flour cost $37\frac{1}{2}$ dollars, what will 1 barrel cost?

15. Divide $15\frac{2}{3}$ by 3.
16. Divide $22\frac{1}{5}$ by 6.
17. Divide $41\frac{3}{4}$ by 11.
18. Divide $65\frac{8}{15}$ by 23.
19. Divide $100\frac{13}{14}$ by 40.
20. Divide $225\frac{17}{25}$ by 50.

CASE II.

To Divide a Whole Number by a Fraction.

1. At $\frac{1}{2}$ dollar apiece, how many chickens can be bought for 3 dollars?

Analysis.—Since 1 half dollar will buy 1 chicken, 3 dollars will buy as many as there are halves in 3 dols., which are 6. Therefore, 3 dols. will buy 6 chickens.

2. At $\frac{1}{8}$ of a dollar a quart, how many quarts of cherries can you buy for 5 dollars?

3. If you divide 8 apples equally among 4 boys, what part, and how many will each receive?

Analysis.—1 is $\frac{1}{4}$ of 4; therefore, each boy will receive $\frac{1}{4}$ part. Again, if 8 apples are divided into 4 equal parts, 1 part will be $\frac{1}{4}$ of 8, which is 2. Therefore, etc.

4. A teacher distributed 16 pounds of figs equally among 5 pupils, what part, and how many did each receive?

5. At $\frac{2}{3}$ of a dollar a yard, how many yards of poplin can be bought for 5 dollars?

Analysis.—In 5 dollars there are 15 thirds, and 2 thirds are contained in 15 thirds, $7\frac{1}{2}$ times. Therefore, etc.

6. At $\frac{3}{4}$ of a cent apiece, how many apples can I buy for 8 cents?

7. At $\frac{3}{8}$ of a dollar a pound, how many pounds of cinnamon can I buy for 10 dollars?

8. At $\frac{7}{8}$ of a dollar a box, how many boxes of white grapes can be bought for 6 dollars?

SLATE EXERCISES.

1. At $\frac{2}{3}$ of a dollar a pound, how many pounds of tea can I buy for 20 dollars?

ANALYSIS.—At 1 third dollar a pound, I can buy as many pounds as there are thirds in 20 dollars, or 60 pounds. But the price is 2 thirds dollar a pound; therefore, I can buy only 1 half of 60 or 30 pounds.

OPERATION.

$$20\text{d.} \div \frac{2}{3} = (20 \times 3) \div 2.$$
$$(20 \times 3) \div 2 = \frac{60}{2}, \text{ or } 30 \text{ p.}$$
$$\text{Or, } 20 \times \frac{3}{2} = \frac{60}{2} = 30 \text{ p.}$$

In the operation we *multiply* the whole number 20 by the denominator 3, and *divide* the *product* by the numerator 2. But this is the same as *inverting* the fractional divisor, and then *multiplying* the dividend by it. (P. 113, Q. 40.)

44. How divide a *whole number* by a *fraction?*

Multiply the whole number by the fraction inverted.

NOTES—1. The *reason* of the rule is this: multiplying the whole number by the given *denominator* reduces it to a fraction having the *same denominator* as the given fraction. Hence, the numerators are *like numbers*, and one may be divided by the other, as whole numbers. (P. 104, Rem.)

2. To divide a *whole* by a mixed number, reduce the *mixed* number to an *improper* fraction. (Ex. 2.)

3. A fraction is *inverted* when its terms are made to *exchange places.* Thus, $\frac{3}{4}$ inverted becomes $\frac{4}{3}$.

2. At $12\frac{1}{2}$ dollars apiece, how many ploughs can a man buy for 75 dollars? *Ans.* 6.

3. Divide 40 by $\frac{3}{4}$.
4. Divide 55 by $\frac{5}{8}$.
5. Divide 68 by $\frac{9}{15}$.
6. Divide 75 by $\frac{7}{18}$.
7. Divide 96 by $18\frac{1}{4}$.
8. Divide 100 by $20\frac{2}{5}$.
9. Divide 250 by $37\frac{1}{2}$.
10. Divide 560 by $66\frac{2}{3}$.

11. A lady paid 62 dollars for $15\frac{1}{2}$ yards of silk: what was the silk a yard?

12. If a horse travels 75 miles in $18\frac{3}{4}$ hours, how far will he go in 1 hour?

CASE III.

To Divide a Fraction by a Fraction when they have a *Common* Denominator.

1. At $\frac{2}{3}$ of a dollar a pound, how many pounds of pepper can be bought for $\frac{7}{3}$ of a dollar?

ANALYSIS.—If 2 thirds of a dollar will buy 1 pound, 7 thirds will buy as many pounds as 2 is contained times in 7, and 2 is contained in 7, $3\frac{1}{2}$ times. Therefore, $\frac{7}{3}$ dollar will buy $3\frac{1}{2}$ pounds.

2. How many needles, at $\frac{2}{5}$ cent apiece, can you buy for $\frac{8}{5}$ cent?

3. How many pen-knives, at $\frac{3}{4}$ of a dollar, can be had for $\frac{12}{4}$ of a dollar?

4. At $\frac{1}{6}$ of a dollar a yard, how many yards of ribbon can be purchased for $\frac{7}{6}$ of a dollar?

5. At $\frac{5}{8}$ of a dollar a yard, how many yards of gingham will $\frac{27}{8}$ of a dollar buy?

SLATE EXERCISES.

REMARK.—When fractions have a *common* denominator, their numerators are *like numbers*. Hence, one *numerator* may be divided by the *other*, as whole numbers.

1. If $\frac{2}{5}$ of a dollar will buy 1 pound of coffee, how many pounds can be bought for $\frac{27}{5}$ of a dollar?

ANALYSIS.—If $\frac{2}{5}$ dollar will buy 1 pound, $\frac{27}{5}$ dollar will buy as many pounds as $\frac{2}{5}$ are contained times in $\frac{27}{5}$, which is $13\frac{1}{2}$ times. Therefore, etc.

OPERATION.

$$\frac{27}{5} \div \frac{2}{5} = 27 \div 2 = 13\frac{1}{2} \text{ p.}$$

45. How divide one fraction by another when they have a common denominator?

Divide the numerator of the dividend by that of the divisor.

2. Divide $\frac{18}{10}$ by $\frac{3}{10}$.
3. Divide $\frac{25}{13}$ by $\frac{6}{13}$.
4. Divide $\frac{43}{17}$ by $\frac{5}{17}$.
5. Divide $\frac{54}{20}$ by $\frac{12}{20}$.
6. Divide $\frac{62}{33}$ by $\frac{13}{33}$.
7. Divide $\frac{70}{35}$ by $\frac{16}{35}$.

To Divide a Fraction by a Fraction, when they have *Different* Denominators.

1. At $\frac{1}{3}$ of a dime apiece, how many pears can be purchased for $\frac{3}{4}$ of a dime?

ANALYSIS.—$\frac{3}{4}$ dime will buy as many pears as $\frac{1}{3}$ dime is contained times in $\frac{3}{4}$ dime. Reducing $\frac{1}{3}$ and $\frac{3}{4}$ to a common denominator, they become $\frac{4}{12}$ and $\frac{9}{12}$. Their numerators are now *like numbers*, and one may be divided by the other. Thus, $9 \div 4 = 2\frac{1}{4}$ pears, the answer required. (P. 120, Rem.)

OPERATION.

$$\frac{1 \times 4}{3 \times 5} = \frac{4}{12};$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12};$$

$$\frac{9}{12} \div \frac{4}{12} = 9 \div 4;$$

$$9 \div 4 = 2\frac{1}{4} \text{ p.}$$

Or, $\frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$, or $2\frac{1}{4}$.

By inspecting the operation of reducing the fractions to a common denominator, it will be seen that the numerator of each is multiplied into the denominator of the other. This produces the *same combination* of terms and the *same results* as *inverting* the divisor and *multiplying* the terms of the dividend by it. Thus, $\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$, or $2\frac{1}{4}$ pears, the same as before.

REMARK.—In dividing, no use is made of the common denominator; hence, in practice, multiplying the denominators together may be omitted.

46. How divide a fraction by a fraction, when they have *different* denominators?

Reduce the fractions to a com. denominator, and divide the numerator of the dividend by that of the divisor.

Or, multiply the dividend by the divisor inverted.

NOTE.—*Mixed* numbers must be reduced to *improper* fractions, and *compound* fractions to *simple* ones. (Ex. 17, 24.)

2. At $\frac{3}{8}$ of a dollar a pound, how much tea can be had for $\frac{4}{5}$ of a dollar?

3. How many pineapples, at $\frac{3}{10}$ of a dollar each, can be had for $\frac{7}{8}$ of a dollar?

4. At $\frac{1}{8}$ of a dollar a pound, how much sugar can be had for $\frac{5}{6}$ of a dollar?

Perform the following divisions:

5. Divide $\frac{2}{3}$ by $\frac{1}{5}$.
6. Divide $\frac{3}{5}$ by $\frac{2}{6}$.
7. Divide $\frac{5}{7}$ by $\frac{2}{3}$.
8. Divide $\frac{7}{10}$ by $\frac{3}{12}$.
9. Divide $\frac{8}{11}$ by $\frac{5}{13}$.
10. Divide $\frac{9}{15}$ by $\frac{3}{25}$.
11. Divide $\frac{11}{15}$ by $\frac{7}{23}$.
12. Divide $\frac{21}{34}$ by $\frac{10}{45}$.
13. Divide $\frac{36}{45}$ by $\frac{8}{25}$.
14. Divide $\frac{48}{100}$ by $\frac{31}{140}$.
15. Divide $\frac{144}{256}$ by $\frac{75}{300}$.
16. Divide $\frac{136}{275}$ by $\frac{83}{284}$.

17. How many bushels of apples, at $2\frac{3}{4}$ dollars a bushel can be purchased with $8\frac{4}{5}$ dollars?

Analysis.—Reducing the *mixed* numbers to *improper* fractions, we have $2\frac{3}{4}=\frac{11}{4}$, $8\frac{4}{5}=\frac{44}{5}$. Inverting the divisor, we cancel the common factor 11, and proceed as before. *Ans.* $3\frac{1}{5}$ bushels.

Operation.

$$2\frac{3}{4}=\frac{11}{4}, \text{ and } 8\frac{4}{5}=\frac{44}{5};$$
$$\frac{44}{5}\div\frac{11}{4}=\frac{44}{5}\times\frac{4}{11};$$
$$\frac{44}{5}\times\frac{4}{11}=\frac{\overset{4}{\cancel{44}}}{5}\times\frac{4}{\cancel{11}}=\frac{16}{5}, \text{ or } 3\frac{1}{5}.$$

18. Divide $3\frac{2}{3}$ by $2\frac{1}{4}$.
19. Divide $8\frac{4}{5}$ by $3\frac{3}{4}$.
20. Divide $13\frac{1}{6}$ by $5\frac{1}{2}$.
21. Divide $18\frac{1}{3}$ by $5\frac{1}{6}$.
22. Divide $27\frac{3}{4}$ by $11\frac{2}{5}$.
23. Divide $55\frac{2}{11}$ by $21\frac{3}{7}$.

24. What is the quotient of $\frac{1}{3}$ of $\frac{3}{5}$ of $3\frac{3}{4}$ divided by $\frac{2}{3}$ of $\frac{3}{4}$?

Analysis.—Reducing $3\frac{3}{4}$ to $\frac{15}{4}$, and inverting the divisor, we have $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{15}{4}\div\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{3}\times\frac{3}{5}\times\frac{15}{4}\times\frac{3}{2}\times\frac{4}{3}=\frac{3}{2}$, or $1\frac{1}{2}$, *Ans.*

25. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{2}{5}$.
26. Divide $\frac{4}{5}$ of $\frac{3}{6}$ of $\frac{5}{8}$ by $\frac{3}{4}$ of $\frac{7}{8}$.
27. Divide $\frac{5}{7}$ of $10\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$.
28. Divide $\frac{1}{9}$ of $\frac{3}{4}$ of $\frac{5}{8}$ by $\frac{1}{3}$ of $6\frac{2}{3}$.

To reduce a Complex Fraction to a Simple one.

1. Reduce the complex fraction $\dfrac{2\frac{1}{3}}{4}$ to a simple one.

Analysis.—The given complex fraction is equivalent to $2\frac{1}{3}\div4$. Reducing the numerator $2\frac{1}{3}$ to a simple fraction, it becomes $\frac{7}{3}$, and $\frac{7}{3}\div4=\frac{7}{12}$, the answer required. (P. 99, Q. 25.)

1st Operation.

$$\frac{2\frac{1}{3}}{4}=2\frac{1}{3}\div4;$$
$$2\frac{1}{3}\div4=\frac{7}{3}\div4;$$
$$\frac{7}{3}\div4=\frac{7}{12}, \textit{ Ans.}$$

Or, reducing both the numerator and denominator to a simple fraction, they become $\frac{7}{3}$ and $\frac{4}{1}$. Now, $\frac{7}{3} \div \frac{4}{1} = \frac{7}{3} \times \frac{1}{4} = \frac{7}{12}$, the same as before.

2D OPERATION.

$2\frac{1}{3} \div 4 = \frac{7}{3} \div \frac{4}{1}$;

$\frac{7}{3} \div \frac{4}{1} = \frac{7}{3} \times \frac{1}{4} = \frac{7}{12}$.

2. Reduce the complex fraction $\dfrac{\frac{1}{2}}{3}$ to a simple one.

SOLUTION.—Performing the division indicated, $\frac{1}{2} \div 3 = \frac{1}{6}$, *Ans.*

47. How reduce a complex fraction to a simple one?

Reduce the numerator to a simple fraction, and divide it by the denominator. (P. 117, Q. 43.)

REMARKS.—1. *Complex* Fractions when reduced to *simple* ones, are *added, subtracted,* etc., like other Simple Fractions.

2. The expressions $\dfrac{2\frac{1}{2}}{5\frac{2}{3}}$, $\dfrac{\frac{3}{4}}{\frac{5}{8}}$, etc., indicate a division of one fractional number by another.

Such expressions are reduced to simple fractions in the same manner as one fraction is divided by another. (Ex. 3.)

3. Reduce the expression $\dfrac{2\frac{1}{2}}{5\frac{2}{3}}$ to a simple fraction.

ANALYSIS.—The given expression is equivalent to $2\frac{1}{2} \div 5\frac{2}{3}$. Reducing the divisor and dividend to simple fractions, they become $\frac{5}{2}$ and $\frac{17}{3}$; and $\frac{5}{2} \div \frac{17}{3} = \frac{5}{2} \times \frac{3}{17} = \frac{15}{34}$. (P. 99, Q. 25.)

OPERATION.

$\dfrac{2\frac{1}{2}}{5\frac{2}{3}} = 2\frac{1}{2} \div 5\frac{2}{3}$;

$2\frac{1}{2} = \frac{5}{2}$, and $5\frac{2}{3} = \frac{17}{3}$;

$\frac{5}{2} \div \frac{17}{3} = \frac{5}{2} \times \frac{3}{17} = \frac{15}{34}$.

Reduce the following complex fractions to simple ones.

4. $\dfrac{\frac{3}{5}}{8}$.	6. $\dfrac{4\frac{1}{2}}{6}$.	8. $\dfrac{\frac{5}{8}}{9}$.	10. $\dfrac{9\frac{1}{4}}{10}$.
5. $\dfrac{7\frac{1}{3}}{7}$.	7. $\dfrac{8\frac{3}{4}}{20}$.	9. $\dfrac{\frac{3}{28}}{35}$.	11. $\dfrac{12\frac{8}{10}}{42}$

48. The preceding principles may be reduced to the following

GENERAL RULE.

Reduce whole and mixed numbers to improper fractions, compound and complex fractions to simple ones, and multiply the dividend by the divisor inverted.

QUESTIONS FOR REVIEW.

1. If you pay $3\frac{1}{4}$ dollars a week for board, what will it cost you to board 11 weeks?

2. If a ton of hay is worth 17 dollars, what is $\frac{3}{4}$ of a ton worth?

3. What will $\frac{5}{8}$ of a yard of ribbon cost, at $\frac{4}{5}$ of a dollar a yard?

4. What cost $10\frac{1}{2}$ pounds of butter, at $\frac{1}{4}$ of a dollar a pound?

5. What cost $16\frac{1}{4}$ yards of silk, at $2\frac{3}{8}$ dollars per yard?

6. At $\frac{4}{5}$ of a dollar a pound, how many pounds of tea can be purchased for 30 dollars?

7. How many pen-knives can I buy for 60 dollars, if I pay $\frac{5}{8}$ of a dollar apiece?

8. At $\frac{2}{5}$ of a dollar a pound, how many pounds of almonds can be bought for $58\frac{3}{4}$ dollars?

9. How much maple sugar, at $\frac{1}{5}$ of a dollar a pound, can be purchased for $\frac{9}{10}$ of a dollar?

10. At $2\frac{1}{4}$ dollars a cord, how much wood can be had for 18 dollars?

11. How much flour, at $7\frac{1}{2}$ dollars a barrel, can be had for $37\frac{1}{2}$ dollars?

12. Required the sum of $\frac{3}{4}$ and $\frac{2}{10}$. Their difference. Their product. The quotient of the former divided by the latter.

13. How many days can you hire a laborer for $37\frac{1}{2}$ dollars, if you pay him $1\frac{3}{4}$ dollar a day?

14. A planter raised 60 bales of cotton, sold $\frac{1}{5}$ of them to one merchant, and $\frac{2}{3}$ to another: how many bales had he left?

15. A speculator bought a quantity of apples for $162\frac{1}{2}$ dollars, and sold them for $210\frac{3}{4}$ dollars: what was his profit?

16. Bought 15 pounds of butter, at $\frac{1}{4}$ dol. a pound; and 10 gal. of molasses, at $\frac{3}{8}$ dol. a gal.: what was the cost of both?

17. At $\frac{1}{5}$ of a dollar a pound, how many raisins can be bought for $\frac{9}{10}$ of a dollar?

18. What is the quotient of $\frac{1}{2}$ of $\frac{4}{5}$ of $5\frac{1}{2}$ divided by $\frac{4}{5}$ of $\frac{3}{4}$?

19. If I pay $\frac{1}{2}$ of $\frac{4}{5}$ of 20 dollars for a ton of coal, what must I pay for $\frac{1}{2}$ of $4\frac{2}{5}$ tons?

20. A man having 500 dollars, laid out $\frac{7}{8}$ of it in cotton, which was $\frac{1}{8}$ of $\frac{5}{8}$ of a dollar a pound: how much cotton did he have?

21. A man owning $\frac{11}{16}$ of a ship, sold $\frac{3}{4}$ of his share of her: what part of the ship did he sell, and what part had he left?

22. How long will 150 pounds of coffee last a family, if they use $3\frac{1}{3}$ pounds a week?

23. A and B drew a prize amounting to $256\frac{1}{2}$ dollars; A took $160\frac{3}{4}$ dollars: how much did B have?

24. What will it cost to build $\frac{1}{2}$ of $\frac{2}{3}$ of $16\frac{1}{2}$ rods of stone wall, at $1\frac{4}{5}$ of a dollar a rod?

25. If $2\frac{7}{8}$ of a yard of velvet can be bought for 12 dollars, what part of a yard can be bought for 1 dollar?

26. Divide $\frac{2}{5}$ of $\frac{3}{4}$ by $\frac{4}{5}$ of $\frac{6}{7}$.

27. Divide $\frac{5}{8}$ of 32 by $\frac{2}{3}$ of $\frac{6}{7}$.

28. Divide $\frac{2}{11}$ of $16\frac{1}{2}$ by $\frac{4}{5}$ of 10.

29. Divide $\frac{7}{8}$ of $\frac{4}{5}$ of $18\frac{2}{3}$ by $\frac{3}{5}$ of $3\frac{1}{2}$.

30. Which will cost more, 8 barrels of flour, at $7\frac{1}{2}$ dollars a barrel, or 16 barrels of potatoes, at $3\frac{1}{4}$ dollars a barrel?

31. If oranges are $6\frac{1}{4}$ cents apiece, how many can be bought for $87\frac{1}{2}$ cents?

32. At $16\frac{2}{3}$ cents a pound, how much lard can be bought for $83\frac{1}{3}$ cents?

FRACTIONAL RELATION OF NUMBERS.

To find what *part* one number is of another.

REMARK.—That numbers may be compared with each other, they must be so far of the same nature that one may properly be said to be a *part* of the other. Thus, a *foot* may be compared with a *yard;* for, one is a *third part* of the other. But it can not be said that a *foot* is a part of a *pound;* therefore the *former* can not be compared with the *latter*.

1. What part of 3 is 1?

ANALYSIS.—If 3 is divided in 3 equal parts, *one* of these parts is 1 third. Therefore, 1 is $\frac{1}{3}$ part of 3.

2. What part of 3 is 2?

ANALYSIS.—2 is 2 times 1 third part of 3, or 2 thirds of 3.

3. In 1 gallon there are 4 quarts: what part of a gallon is 1 quart? What part is 3 quarts?

4. What part of 5 is 3? Is 4? Is 2? Is 1?

5. What part of 6 is 2? Is 3? Is 4? Is 5?

6. What part of 4 apples are 5 apples?

ANALYSIS.—1 apple is 1 fourth part of 4 apples; therefore, 5 apples must be 5 times 1 fourth, or 5 fourths of 4 apples.

7. What part of 8 pounds is 9 pounds? Is 11 pounds?

SLATE EXERCISES.

1. What part of 5 cents is 3 cents?

ANALYSIS.—3 cents are 3 times $\frac{1}{5}$, or $\frac{3}{5}$ of 5 cents.

49. How find what part one number is of another?

Make the number denoting the part the numerator, and that with which it is compared the denominator.

NOTE.—The fraction thus found should be reduced to its lowest terms.

2. What part of 48 is 12? Of 63 is 28?

3. What part of 81 is 27? Of 90 is 63? Of 100 is 40?

4. What part of 35 dollars is 19 dollars?

5. If I divide a bushel of plums equally among 15 boys, what part of a bushel will 1 boy receive? What part will 9 boys receive?

6. Helen's age is 18 years, and her brother's 14: what part of her age is her brother's?

7. Henry has 91 marbles, and Charles 70: Charles' marbles are equal to what part of Henry's?

8. If 5 pencils cost 17 cents, what will 4 pencils cost?

ANALYSIS.—4 pencils are $\frac{4}{5}$ of 5 pencils; therefore, 4 pencils will cost $\frac{4}{5}$ of 17 cents. Now, $\frac{1}{5}$ of 17 cents is $3\frac{2}{5}$ cents, and 4 fifths are 4 times $3\frac{2}{5}$ cents, or $13\frac{3}{5}$ cents, the cost required.

$$\begin{array}{r} 5\,)\,17 \text{ cts.} \\ \hline 3\frac{2}{5} \\ 4 \\ \hline \textit{Ans. } 13\frac{3}{5} \text{ cts.} \end{array}$$

9. If 8 oranges cost 32 cents, what will 6 cost?

10. If 20 cows cost 625 dollars, what will 35 cost?

11. If 13 sofas cost 572 dollars, what will 6 cost?

12. What part of 4 pears is $\frac{2}{3}$ of a pear?

ANALYSIS.—1 pear is $\frac{1}{4}$ part of 4 pears; hence, $\frac{2}{3}$ of a pear must be $\frac{2}{3}$ of $\frac{1}{4}=\frac{2}{12}$, or $\frac{1}{6}$ of a pear. Therefore, etc.

OPERATION.

$$\frac{\frac{2}{3}}{4}=\frac{2}{3}\div 4;$$

$$\frac{2}{3}\div 4=\frac{2}{12}, \text{ or } \frac{1}{6}.$$

Making the *fraction* which denotes the part the *numerator*, and the *whole number* the *denominator*, we have the complex fraction $\frac{\frac{2}{3}}{4}$, to be reduced to a simple one. (P. 123, Q. 47.)

Or, what is the same thing, a fraction to be divided by a whole number; and $\frac{2}{3}\div 4=\frac{2}{12}$, or $\frac{1}{6}$. (P. 117, Q. 43.)

13. What part of 15 is $\frac{3}{7}$? 15. What part of 45 is $\frac{9}{10}$?

14. What part of 26 is $\frac{5}{8}$? 16. What part of 63 is $\frac{8}{15}$?

17. If 15 barrels of flour cost 100 dollars, what will $\frac{3}{4}$ of a barrel come to?

18. If 20 acres of land yield 250 bushels of corn, what will $\frac{7}{8}$ of an acre yield?

To find a number when a *part* of it is given.

1. 5 is $\frac{1}{3}$ of what number?

ANALYSIS.—If 5 is $\frac{1}{3}$, 3 thirds, or the whole number, must be 3 times 5, or 15. Therefore, 5 is $\frac{1}{3}$ of 15.

2. 6 is $\frac{1}{4}$ of what number? 7 is $\frac{1}{8}$ of what number?

3. 8 is $\frac{2}{3}$ of what number?

ANALYSIS.—Since 8 is $\frac{2}{3}$ of a certain number, 1 third is $\frac{1}{2}$ of 8, which is 4, and 3 thirds must be 3 times 4, or 12. Therefore, etc.

5. George has 12 apples, which are $\frac{3}{4}$ of the number which William has: how many has William?

SLATE EXERCISES.

1. 16 is $\frac{2}{3}$ of what number?

1ST ANALYSIS.—Since $\frac{2}{3}$ of a number is 16, $\frac{3}{3}$ or the whole number, must be as many units as $\frac{2}{3}$ is contained times in 16; and $16 \div \frac{2}{3} = 16 \times \frac{3}{2}$, or 24.

1ST OPERATION.

$$16 \div \tfrac{2}{3} = 16 \times \tfrac{3}{2};$$
$$16 \times \tfrac{3}{2} = \tfrac{48}{2}, \text{ or } 24.$$

2D ANALYSIS.—Since 16 is $\frac{2}{3}$ of a certain number, 1 third of that number must be $\frac{1}{2}$ of 16, which is 8. 3 thirds, or the whole number must be 3 times 8, or 24.

2D OPERATION.

$$\tfrac{1}{3} = 16 \div 2 = 8;$$
$$\tfrac{3}{3} = 8 \times 3 = 24.$$

50. How find a number when a part of it is given?

Divide the number denoting the part by the fraction.

Or, find one part as indicated by the numerator of the fraction, and multiply this by the denominator.

2. 32 is $\frac{3}{4}$ of what?
3. 45 is $\frac{4}{5}$ of what?
4. 72 is $\frac{5}{6}$ of what?
5. 100 is $\frac{5}{9}$ of what?
6. 144 is $\frac{16}{29}$ of what?
7. 250 is $\frac{10}{35}$ of what?

8. If $\frac{7}{8}$ of an acre of land is worth 35 dollars, what is a whole acre worth?

9. A man paid 75 dollars toward a horse, which was $\frac{7}{15}$ of the price: what did he give for the horse?

10. A man being asked how old he was, replied that $\frac{7}{12}$ of his age equaled 49 years: what was his age?

DECIMAL FRACTIONS.

PRELIMINARY EXERCISES.

1. If a sheet of paper is divided into 10 equal parts, what part of a sheet is 1 of these parts?

One of these parts is $\frac{1}{10}$ of a sheet.

2. If one of these *tenths* is divided into 10 other equal parts, what part of a sheet is 1 of these parts?

One of these parts is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$ of a sheet.

3. If one of these *hundredths* is divided into 10 other equal parts, what part of a sheet is 1 of these parts?

Each part is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{1000}$ of a sheet.

4. What is meant by a *tenth*, a *hundredth*, a *thousandth*, etc.?

A *tenth* is one of the *ten equal parts* into which a number or thing may be divided, etc.?

5. How much *greater* are tens than units; hundreds than tens; thousands than hundreds, etc.?

Tens are 10 times *greater* than *units*, and each succeeding order is 10 times *greater* than the preceding.

6. How much *less* are tenths than units; hundredths than tenths; thousandths than hundredths, etc.?

Tenths are 10 times *less* than *units; hundredths* are 10 times *less* than *tenths;* and so on, each *succeeding* order being 10 times *less* than the *preceding.*

7. What places do tens, hundreds, thousands, etc., occupy?

Tens occupy the *first* place on the left of units; *hundreds,* the *second; thousands,* the *third,* etc.

8. Following this analogy, what place should *tenths, hundredths, thousandths,* etc., occupy?

Tenths in the decreasing scale correspond with *tens* in the increasing scale; hence they should occupy the *first* place on the *right* of units. In like manner, *hundredths,* which correspond with *hundreds,* should occupy the *second* place; *thousandths,* the *third* place, etc.

9. How many units make a ten, tens a hundred, etc.?

10. How do the orders of whole numbers increase?

They increase from *right* to *left* by the *scale* of 10.

11. How many tenths make a unit; hundredths a tenth; thousandths a hundredth, etc.

Ten tenths make a *unit; ten hundredths* make a *tenth; ten thousandths* make a *hundredth*, etc.

12. How do the orders of these fractions decrease?

They decrease from *left* to *right* by the *scale* of 10.

NOTATION OF DECIMALS.

13. What are Decimal Fractions?

Decimal Fractions are those in which the *unit* is divided into *tenths, hundredths, thousandths*, etc.

They arise from dividing a unit into *ten equal* parts, or *tenths;* then subdividing one of these tenths into *ten* other equal parts, or *hundredths;* and so on, the successive orders decreasing regularly by the *scale* of 10.

14. How, and upon what principle are they expressed?

By placing a *point* before the *numerator*, and assigning to each figure a *value* according to the *place* it *occupies*, as in whole numbers. Thus, $\frac{3}{10}$ is expressed by writing 3 in the *first* place on the right of units; as, .3; $\frac{3}{100}$ by writing 3 in the *second* place; as, .03; $\frac{3}{1000}$ by writing 3 in the *third* place; as, .003.

15. What do figures standing in the *first, second, third,* etc., places on the right of units denote?

When standing in the *first* place on the right of units, they denote *tenths;* in the *second* place, they denote *hundredths;* in the *third* place, *thousandths*, etc.

NOTES.—1. The *point* used to distinguish *decimals* from *whole* numbers, is called the *decimal point*.

2. These fractions are called *decimals* from the Latin *decem*, *ten*, which indicates their *origin* and *scale* of *decrease*.

16. **What is the denominator of a decimal fraction?**

It is always 10, 100, 1000, etc., or 1 with as *many ciphers* annexed as there are *decimals* in the *numerator.*

Name the orders of integers, beginning at *units.*

Name the orders of decimals, beginning at *units place.*

TABLE.

Millions, etc.	Hundreds of thousands.	Tens of thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*	(Decimal Point.)	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hundred-thousandths.	*Millionths, etc.*
6	5	2	3	4	7	3	.	5	2	8	7	3	5

Integers. Decimals.

17. **What is the effect of *prefixing* ciphers to decimals?**

Each cipher prefixed to a decimal, *diminishes* its value *ten times,* or *divides* it by 10.

18. **What is the effect of *annexing* ciphers to decimals?**

The *value* is *not altered.* Thus, .3=.30=.300, etc.

19. **How write decimals?**

Write the figures of the numerator in their order, assigning to each its proper place below units, and prefix to them the decimal point.

If the numerator has not as many figures as required, supply the deficiency by prefixing ciphers.

NOTE.—A *decimal* and *integer* written together, are called a *mixed number;* as, 35.263. (P. 92, Q. 13.)

1. On which side of units are tens? Tenths? Thousands? Hundredths? Hundreds? Thousandths?

2. What is the name of the second place on the right of units? The fourth? The third? The fifth?

3. How many decimal places are required to express tenths? Thousandths? Hundredths? Millionths?

SLATE EXERCISES.

Write the following fractions decimally:

1. 12 hundredths.	7. 9 thousandths.
2. 25 hundredths.	8. 13 thousandths.
3. 5 hundredths.	9. $\frac{1345}{10000}$.
4. 49 hundredths.	10. $\frac{236}{10000}$.
5. 119 thousandths.	11. $\frac{39}{10000}$.
6. 27 thousandths.	12. $\frac{7}{100000}$.

13. Write 6 hundredths. 41 thousandths. 7 thousandths.

14. Write 201 ten-thousandths. 752 hundred-thousandths.

15. Write 5 millionths. 63 millionths. 98 millionths. 375 millionths.

20. How read decimals?

Read the decimals as whole numbers, and apply to them the name of the lowest order.

REMARK.—The *unit's place* should always be the *starting point* both in reading and writing decimals.

Copy and read the following:

15. .7.	21. 2.35.	27. 21.251.	33. 121.4502.
16. .75.	22. 3.236.	28. 30.4312.	34. 240.4023.
17. .06.	23. 5.078.	29. 44.0643.	35. 306.46531.
18. .121.	24. 6.2356.	30. 53.21034.	36. 500.00729.
19. .065.	25. 7.3062.	31. 72.05213.	37. 607.329267.
20. .008.	26. 8.5602.	32. 84.00605.	38. 730.004308.

*** Dictation exercises in reading and writing decimals should be practiced till the class is perfectly familiar with them.

REDUCTION OF DECIMALS.

CASE I.

To Reduce Decimals to Common Fractions.

1. Reduce .27 to a common fraction.

OPERATION.

$.27=\frac{27}{100}$, *Ans.*

ANALYSIS.—Since .27 has two decimal figures, its denominator must be 100. Hence, $.27=\frac{27}{100}$. In the operation we omit the decimal point, and place the denominator 100 under the 27.

21. How reduce decimals to common fractions?

Erase the decimal point, and place the denominator under the numerator. (P. 131, Q. 16.)

NOTE.—After decimals are reduced to common fractions, they may be reduced to *lower terms*, to a *common denominator*, etc., and then be treated in all respects like other *common* fractions.

2. Reduce .35 to a common fraction, and to its lowest terms. *Ans.* $.35=\frac{35}{100}$, and $\frac{35}{100}=\frac{7}{20}$.

Reduce the following decimals to common fractions in their lowest terms:

3. .24.	7. .04.	11. .4032.	15. .00045.
4. .135.	8. .025.	12. .0005.	16. .00328.
5. .404.	9. .204.	13. .0106.	17. .01032.
6. .675.	10. .1025.	14. .7524.	18. .123456.

CASE II.

To Reduce Common Fractions to Decimals.

1. Reduce $\frac{1}{4}$ to a decimal.

OPERATION.

$$4\,)\,\underline{1.00}$$
$$.25$$

ANALYSIS.—$\frac{1}{4}$ is equivalent to 1 divided by 4. But 1 cannot be divided by 4; we therefore reduce it to *tenths* by annexing a cipher to it, making 10 tenths. (P. 57, Q. 18.) Now $\frac{1}{4}$ of 10 tenths=2 tenths and 2 tenths over. Reducing the 2 tenths to *hundredths* by annexing a cipher, we have 20 hundredths; and $\frac{1}{4}$ of 20 hundredths=5 hundredths. Therefore, $\frac{1}{4}$ equals .25.

22. How reduce common fractions to decimals?

Annex ciphers to the numerator, and divide by the denominator.

Finally, point off as many decimal figures in the result as there are ciphers annexed to the numerator.

Reduce the following fractions to decimals:

2. $\frac{1}{2}$.	6. $\frac{3}{6}$.	10. $\frac{9}{10}$.	14. $\frac{5}{80}$.
3. $\frac{1}{5}$.	7. $\frac{3}{8}$.	11. $\frac{15}{20}$.	15. $\frac{3}{120}$.
4. $\frac{3}{4}$.	8. $\frac{4}{10}$.	12. $\frac{5}{16}$.	16. $\frac{25}{250}$.
5. $\frac{2}{6}$.	9. $\frac{5}{12}$.	13. $\frac{7}{35}$.	17. $\frac{100}{500}$.

18. Reduce $\frac{1}{3}$ to the form of a decimal.

ANALYSIS.—Annexing ciphers to the numerator, and dividing by the denominator as before, the quotient is 3 repeated continually, and the remainder is always 1. Hence, $\frac{1}{3}$ cannot be exactly expressed by decimals.

OPERATION.

$$3\,)\,\underline{1.0000}$$
$$.3333, \text{ etc.}$$

19. Reduce $\frac{15}{33}$ to the form of a decimal.

ANALYSIS.—Annexing ciphers and dividing as before, the quotient is 45 repeated continually, and the remainder is alternately 18 and 15, the latter being the given numerator. Therefore, $\frac{15}{33}$ cannot be exactly expressed by decimals.

OPERATION.

$$33\,)\,\underline{15.0000}$$
$$.4545, \text{ etc.}$$

23. When the numerator with ciphers annexed is *exactly* divisible by the denominator, what is the decimal called?

It is called a *Terminate decimal.*

24. When it is *not exactly* divisible, and the same figure or set of figures continually recurs in the quotient, what is the decimal called?

It is called an *Interminate,* or *Circulating decimal.*

25. What are the figure or figures repeated called?

They are called the *Repetend.*

NOTE.—When the quotient has been carried as far as desirable, the sign (+) is annexed to it, to indicate there is still a *remainder.*

ADDITION OF DECIMALS.

1. If an arithmetic costs 6 tenths dollar, and a grammar 8 tenths dollar, what will both cost?

ANALYSIS.—Since each of these decimals expresses *tenths* they have a *common denominator*, viz., 10; therefore, they are *like numbers*, and may be added, as *whole* numbers. Now 6 tenths and 8 tenths are 14 tenths; equal to 1 and 4 tenths dollar

25, *a*. When have decimals a common denominator?

Decimals have a common denominator when their numerators have the same number of decimal figures. As .05 and .07, whose denominator is 100. (P. 131, Q. 16.)

25, *b*. How reduce decimals to a common denominator?

Make the number of decimal figures the same in each, by annexing ciphers. (P. 131, Q. 18.) Thus, .3 and .05 reduced to a common denominator become .30 and .05.

1. What is the sum of 42.136; 6.35; 13.7; and .245?

ANALYSIS.—Reduce the decimals to a common denominator by annexing ciphers, or, which is the same, write units under units, tenths under tenths, etc., the decimal points being in a perpendicular line. Beginning at the right, add as in whole numbers, and place the decimal point in the *amount* under those in the numbers added. (P. 25, Q. 9.)

OPERATION.

	42.136
	6.350
	13.700
	.245
Ans.	62.431

26. How add decimals?

I. *Write the numbers so that the decimal points shall stand one under another, with tenths under tenths, etc.*

II. *Beginning at the right, add as in whole numbers, and place the decimal point in the amount under those in the numbers added.* (P. 28, Q. 13.)

REM.—Placing *tenths* under *tenths*, *hundredths* under *hundredths*, etc., in effect reduces decimals to a *common denominator*; hence, the *ciphers* on the right may be omitted in the operation. (P. 131, Q. 18.)

(2.)	(3.)	(4.)	(5.)
26.176	8.65	206.451	3.7056
2.5	.372	40.45	.045
4.38	1.6	3.6	.06
.023	5.405	23.75	2.841

6. What is the sum of seventeen and four tenths; si.. and two hundredths; eight and forty-five thousandths?

7. What is the sum of 13.71 yards; 21.2 yards; and 10.75 yards?

8. How many dollars in 3 purses; the first containing 26.5 dollars; the second, 17.25 dollars; and the third, 30.625 dollars?

9. How many acres in four lots, which respectively contain 19.275 acres; 30.41 acres; 23.261 acres; and 31.027 acres?

10. What is the sum of 42.07 gallons + 50.128 gals. + 1.625 gals. + 16.018 gals.?

11. What is the sum of 28.16 rods + 45.025 rods + 85.7 rods + 17.265 rods.

SUBTRACTION OF DECIMALS.

1. A lad having 8 tenths of a dollar, paid 3 tenths of dollar for his lunch: how much had he left?

ANALYSIS.—3 tenths from 8 tenths leave 5 tenths. Therefore &c.

2. Take 7 tenths from 9 tenths.

3. What is the difference between .19 and .17?

4. Paid .37 dol. for a bushel of apples, and .60 dol. for a bushel of corn: what was the difference in price?

5. A man having .75 of an acre of land, sold .48 of an acre: how much land did he have left?

6. What is the difference between .93 and .62?

SLATE EXERCISES.

1. What is the difference between 2.34 and .543?

	OPERATION.
ANALYSIS.—Reduce the decimals to a common denominator by annexing ciphers, or by writing *units* under *units*, *tenths* under *tenths*, etc., the decimal points being in a perpendicular line. (P. 38, Q. 9.)	2.34 .543 1.797

Beginning at the right, we see that 3 thousandths can not be taken from 0 thousandths; hence we borrow 10, and proceed as in whole numbers.

27. How subtract decimals?

I. *Write the less number under the greater, so that the decimal points shall stand one under the other, with tenths under tenths, etc.*

II. *Beginning at the right, subtract as in whole numbers, and place the decimal point in the remainder under that in the subtrahend.* (P. 42, Q. 15.)

	(2.)	(3.)	(4.)	(5.)
From	6.432	13.206	28.3607	1.00042
Take	3.17	7.0378	.981	.236

Perform the subtractions indicated in the following:

6. 63.025 — 13.5.
7. 7.46 — 3.678.
8. 100.007 — 0.845.
9. 275—60.75.
10. 17.4 — 10.0008.
11. 60.001 — 45.008.
12. 1.0006 — 0.37.
13. 0.05 — 0.005.
14. 0.006 — 0.0006.
15. 0.0001 — .00001.

16. Sold 2 pieces of cloth, one 37.5 yards long, the other 31¼ yards: what was the difference in their length?

17. A man owning 7 tenths of a ship, sold 25 hundredths of her: how much had he left?

18. If from 150.05 acres of land, 87¾ acres are taken, how much will be left?

MULTIPLICATION OF DECIMALS.

1. At .5 of a cent apiece, what will 7 apples come to?

ANALYSIS.—Since 1 apple costs 5 tenths of a cent, 7 apples will cost 7 times 5 tenths, or 35 tenths of a cent; and 35 tenths are equal to 3.5 cents. Therefore, etc.

2. What cost 4 oranges, at .6 of a dime apiece?
3. At .3 dollar apiece, what will 6 melons cost?
4. How many tenths are 5 times 7 tenths? 6 times 4 tenths?
5. How many hundredths are 3 times .15?
6. How many tenths in 4 times .25?

SLATE EXERCISES.

1. If 1 yard of muslin costs .25 dollar, what will 7 yards amount to?

ANALYSIS.—.25 are equal to 2 tenths and 5 hundredths. Now 7 times 5 hundredths are 35 hundredths, equal to 3 tenths and 5 hundredths. Set the 5 in hundredths place, and carry the 3 to the product of tenths. 7 times 2 tenths are 14 tenths, and 3 are 17 tenths, equal to 1 unit and 7 tenths. Write the 7 in tenths place, and the 1 in units place.

OPERATION.

$$\begin{array}{r} .25 \text{ dol.} \\ \underline{7} \\ 1.75 \text{ dol.} \end{array}$$

2. Multiply .375 dollar by 5. *Ans.* 1.875 dol.
3. At .75 dollar a yard, what cost .5 yard of delaine?

ANALYSIS.—$.75=\frac{75}{100}$, and $.5=\frac{5}{10}$. (P. 133, Q. 21.) Now $\frac{5}{10}$ times $\frac{75}{100}=\frac{375}{1000}=375\div1000$, or .375, *Ans.* Instead of multiplying $\frac{75}{100}$ by $\frac{5}{10}$, in the operation we multiply the decimals as whole numbers; consequently the product is as many times too large as there are units in the product of their denominators, viz., 1000. To correct this, we point off 3 figures on the right of the product, which divides it by 1000.

OPERATION.

$$\begin{array}{r} .75 \text{ dol.} \\ \underline{.5} \\ \textit{Ans. } .375 \text{ dol.} \end{array}$$

By inspecting these operations, it will be seen that each *product* has as many decimal figures as both its factors.

In like manner it may be shown, that the *product* of *any two decimals* must have as many decimal figures as both factors.

28. How multiply decimals?

Multiply as in whole numbers, and from the right of the product, point off as many figures for decimals as there are decimal places in both factors.

REMARKS.—1. If the product has not as many figures as there are decimals in both factors, the deficiency must be supplied by prefixing ciphers. (Ex 4.)

2. To multiply a decimal by 10, 100, 1000, etc., *remove the decimal point* as many figures to the *right* as there are ciphers in the multiplier. For, each removal of the decimal point *one* place to the right, multiplies the number by 10.

4. What is the product of .004 multiplied by .03?

ANALYSIS.—The product of the significant figures 4 × 3 = 12. But there are five decimals in the given factors; therefore the product must have five decimals. Prefixing three ciphers to 12, it becomes .00012.

OPERATION.

$$\begin{array}{r} .004 \\ .03 \\ \hline .00012 \end{array}$$ *Ans.*

	(5.)	(6.)	(7.)	(8.)
Mult.	.127	3.025	.0046	250.07
By	.03	.012	.23	3.04

Perform the following multiplications:

9. 8.02 × 3.2.	13. 38.005 × .003.	17. 25.012 × 2.15.
10. 3.51 × .09.	14. 506.12 × .016.	18. 10000 × .007.
11. 9.027 × 13.	15. 407.01 × .123.	19. 000.01 × 300.
12. 365 × .05.	16. 1.0004 × .006.	20. 0.0004 × 2.01.

21. Allowing 5.5 yards to a rod, how many yards are there in 20.25 rods?

22. If a man earns 1.25 dol. a day, how much will he earn in 19.5 days?

23. How many pounds of coffee in 10½ sacks, allowing 37.5 pounds to a sack?

24. If a gallon of molasses is worth .54 dol., how much are 18.75 gallons worth?

25. What is the product of 1.005 multiplied by .008?

26. What is the product of one thousandth into seven hundredths?

27. What is the product of five ten-thousandths into seven tenths?

DIVISION OF DECIMALS.

MENTAL EXERCISES.

1. At 2 tenths of a dime apiece, how many oranges can a lad buy for 8 tenths of a dime?

ANALYSIS.—2 tenths dime are contained in 8 tenths dime, 4 times. Therefore, etc. (P. 63, Q. 10.)

2. How many times 3 tenths in 6 tenths of a dollar?

3. How many times are 7 hundredths contained in 35 hundredths? .4 in .8? $\frac{5}{100}$ in $\frac{25}{100}$?

4. If a man pays 6 tenths of a dollar for 2 tenths of a barrel of apples, what must he pay for 1 tenth of a barrel?

ANALYSIS.—The object is to divide 6 into 2 equal parts. (P. 63, Q. 10.) Since these fractions have a common denominator, one numerator may be divided by the other like whole numbers.

5. How many times are .08 contained in .64?

6. Divide .4 by .2; .6 by .3; .8 by .4.

7. Divide .08 by .02; .16 by .04.

SLATE EXERCISES.

1. How many times .2 in .6?

ANALYSIS.—Since these decimals have a common denominator, they are *like numbers*; hence, one can be divided by the other as common fractions, and the quotient is a whole number. (Page 121, Q. 46.)

OPERATION.

.2).6

Ans. 3 times.

2. How many times .04 in .3?

ANALYSIS.—Reducing the given decimals to a common denominator, we have .04 and .30. Now, .04 is in .30, 7 times and .02 remainder. We put the 7 in *units'* place, because it is *units*. Annexing a cipher to the remainder, .04 is in .020, .5 times and 0 remainder. We set the 5 in *tenths'* place. *Ans.* 7.5 times.

OPERATION.

$$.04\overline{).30}$$
Ans. 7.5

3. How many times .4 in .012?

ANALYSIS.—Reducing these decimals to a common denominator, we have .4 = .400 and .012. Now, as .400 is not contained in .012, we put a cipher in units' place. Annexing a cipher to the dividend, we find .400 is not contained in .0120; we therefore put a cipher in tenths' place. Annexing another cipher, .400 is in .01200, 3 hundredths time and 0 remainder. *Ans.* .03 times.

OPERATION.

$$.400\overline{).012}$$
Ans. 0.03

29. How are decimals divided?

Reduce the decimals to a common denominator, and divide the numerator of the dividend by that of the divisor, placing a decimal point on the right of the quotient.

Annex ciphers to the remainder, and divide as before. The figures on the left of the decimal point denote whole numbers; those on the right, decimals.

Or, divide as in whole numbers, and from the right of the quotient, point off as many decimals as the decimal places in the dividend exceed those in the divisor.

REMARKS.—1. If there are not figures enough in the quotient for the decimals required by the second method, *prefix* ciphers.

2. To divide a decimal by 10, 100, 1000, etc.,

Remove the decimal point in the dividend as many places to the *left* as *there are ciphers* in the divisor.

3. If there is a remainder after carrying the work as far as desired, the sign (+) is annexed to the quotient to show it is not exact.

EXAMPLES FOR PRACTICE.

1. What is the quotient of .028 divided by 7? *Ans.* .004.
2. What is the quotient of .432 divided by .144? *Ans.* 3.
3. What is the quotient of 515 divided by 1.03? *Ans.* .500.

4. Divide 2.37 by 9. *Ans.* 0.2633 + ; or, .2633⅓.

5. At .25 dol. a pound, how much honey can be bought for 2.75 dollars?

6. How many building-lots can be made from 12.75 acres of land, allowing .25 of an acre to a lot?

7. Divide 43.12 by 10.

8. Divide 7.312 by 100.

9. Divide .2806 by 1000.

10. Divide 734.201 by 10000.

Perform the following divisions:

11. $57 \div .4$.
12. $19 \div .25$.
13. $.675 \div .33$.
14. $.0342 \div .07$.
15. $.0039 \div .26$.
16. $36.5 \div 10$.
17. $3.85 \div 100$.
18. $.056 \div .112$.
19. $391.04 \div 1000$.
20. $246.751 \div 85$.
21. $10 \div .01$.
22. $11 \div .11$.
23. $.11 \div 11$.
24. $.0005 \div 5$.
25. $.00003 \div .00004$.

26. A farmer sold 75 sheep for 187.5 dollars: what was that apiece?

27. If you travel 40.75 miles in a day, how long will it take to travel 195.6 miles?

28. If 34.5 bushels of apples cost 17.25 dollars, what will 1 bushel cost?

29. If 18.75 tons of hay cost 196.875 dollars, what will 1 ton cost?

30. How many revolutions will a wheel 9.4 ft. in circumference make in going 5280 feet?

31. If 1 acre of land produces 25.6 bushels of corn, how many acres will be required to produce 4635 bushels?

32. How many times are five thousandths contained in 37 hundredths?

33. How many times are seventy-five ten-thousandths contained in eight thousandths.

34. How many times are seven millionths contained in three hundred-thousandths?

UNITED STATES MONEY.

1. What is Money?

Money is the *standard of value*, and is often called *Currency.*

2. What is United States Money?

United States Money is the national currency of the United States. It is also called *Federal Money.*

3. What are its denominations?

Eagles, dollars, dimes, cents, and *mills.*

TABLE.

10 mills (*m.*)	make	1 cent,	*ct.*
10 cents	"	1 dime,	*d.*
10 d., or 100 cts.	"	1 dollar,	$, or *dol.*
10 dollars	"	1 eagle,	*E.*

50 cts.$=\frac{1}{2}$ dol.; $33\frac{1}{3}$ cts.$=\frac{1}{3}$ dol.; 25 cts.$=\frac{1}{4}$ dol.;
20 cts.$=\frac{1}{5}$ dol.; $12\frac{1}{2}$ cts.$=\frac{1}{8}$ dol.; 10 cts.$=\frac{1}{10}$ dol.

NOTES.—1. It will be observed that the *denominations* of U. S. money, like the *orders* of whole numbers, *increase* and *decrease* by the *scale* of 10. It is thence called *Decimal Currency.*

2. The *sign* of U. S. money is the character ($), called the *dollar* mark, placed before the sum to be expressed.

4. How is U. S. Money written?

Dollars are written as *whole numbers*, with the ign ($) prefixed to them.

Cents are written in the *first two* places on the right of the *decimal point;* because they are *hundredths* of a dollar. Thus 13 dollars 25 cents are written $13.25.

Mills are written in the *third* place on the right; because they are *thousandths* of a dollar; as $25.038.

REMARKS.—1. Eagles are expressed by *tens of dollars;* dimes by *tens of cents.* Thus, 15 eagles are $150, and 6 dimes are 60 cents.

2. As *cents* occupy *two* places, if the number to be expressed is *less* than 10, a *cipher* must be prefixed to the figure denoting them.

3. In business calculations, if the mills in the *result* are 5 or more, they are considered a *cent*; if less than 5, they are omitted.

1. Write 17 dollars and 5 cents. *Ans.* \$17.05.
2. Write 20 dollars, 10 cents, and 3 mills. *Ans.* \$20.103.
3. Express 3 eagles and 4 dimes, in dollars and cents.

ANALYSIS.—Since in 1 eagle there are 10 dollars, in 3 E. there are 3 times 10, or \$30. Again, in 1 dime there are 10 cents, and in 4 dimes 4 times 10, or 40 cents. *Ans.* \$30.40.

4. Write 43 dollars, 12 cents and 5 mills.
5. Write 100 dollars and 8 cents.
6. Write 219 dollars, 3 cents and 4 mills.
7. Write a thousand and ten dollars and five cents.

EXERCISES IN READING U. S. MONEY.

5. How read U. S. Money?

Read the figures on the left of the decimal point, as dollars; those in the first two places on the right, as cents; the next one, as mills; the others, as decimals of a mill.

Copy and read the following sums of U. S. Money?

1. \$17.213.	6. \$100.	11. \$1000.043.
2. \$30.105.	7. \$107.	12. \$2100.05.
3. \$42.60.	8. \$110.50.	13. \$1006.40.
4. \$0.437.	9. \$230.061.	14. \$3050.10.
5. \$0.805.	10. \$500.007.	15. \$4100.01

REDUCTION OF U. S. MONEY.

CASE I.

To Reduce Dollars to Cents and Mills.

1. How many cents are there in \$4?

ANALYSIS.—Since there are 100 cents in a dollar, there must be 100 times as many cents as dollars, or 400 cts. (P. 56, Q. 17.)

2. In \$6, how many cents? In \$7? In \$10? In 12?

3. How many mills in \$6?

ANALYSIS.—There are 1000 mills in a dollar; hence, in \$6 there must be 1000 times as many mills as dols., or 6000 mills.

4. How many mills in 15 cents? In 52 cents?

5. In \$7, how many mills? In \$11? In \$20?

SLATE EXERCISES.

1. How many cents in 18 dollars?

ANALYSIS.—Since there are 100 cents in a dollar, there must be 100 times as many cents as dollars; and 100 times 18 are 1800. Therefore, etc.

OPERATION.

18 dollars.
100
Ans. 1800 cts.

2. In 87 cents, how many mills?

ANALYSIS.—Since there are 10 mills in a cent, there must be 10 times as many mills as cents; and 10 times 87 are 870.

87 cents.
10
Ans. 870 mills.

6. How reduce dollars to cents, etc.?

To reduce *dollars* to *cents, multiply them by* 100.

To reduce *dollars* to *mills, multiply them by* 1000.

To reduce *cents* to *mills, multiply them by* 10.

NOTE.—To reduce dollars, cents, and mills, to *cents* and *mills*, *erase* the *sign of dollars* (\$) and the *decimal point.*

3. In \$40.75, how many cents? *Ans.* 4075 cents.

4. In \$51.073, how many mills? *Ans.* 51073 mills.

Reduce the following to the denominations indicated?

5. \$67 to cents.
6. \$125 to cents.
7. \$95 to mills.
8. \$216 to mills.
9. \$46.10 to cents.
10. \$85.38 to cents.
11. \$7.375 to mills.
12. \$9.87½ to mills.
13. \$537 to cents.
14. \$1385 to mills.

CASE II.

To reduce Cents and Mills to Dollars.

1. How many dollars in 212 cents?

ANALYSIS.—Since in 100 cents there is $1, in 212 cents there are as many dollars as 100 is contained times in 212; and 100 is contained in 212, 2 times and 12 cents over. Therefore, etc.

2. How many dollars in 500 cents? In 625 cents?
3. How many dollars in 700 cents? In 865 cents?
4. How many dollars in 3000 mills?

SOLUTION.—As many as 1000 is contained times in 3000, which is 3 times.

5. How many dollars in 5256 mills? In 7341 mills?
6. How many cents in 327 mills? In 432 mills?

SLATE EXERCISES.

1. How many dollars in 348 cents?

ANALYSIS.—Since in 100 cents there is 1 dollar, in 348 cents there are as many dollars as there are times 100 cents in 348 cents; and 100 is in 348 cents, 3 times and 48 cents over. Therefore, etc.

OPERATION.

$$\begin{array}{r} 1|00)\ 3|48 \\ \hline \$3.48 \end{array}$$

2. How many dollars in 4285 mills?

SOLUTION.—We divide the given mills by 1000, or what is the same thing, cut off 3 figures on the right of the dividend.

$$\begin{array}{r} 1|000)\ 4|285 \\ \hline \textit{Ans. } \$4.285 \end{array}$$

7. How reduce cents and mills to dollars?

To reduce cents to dollars, *divide them by* 100.

To reduce mills to dollars, *divide them by* 1000.

To reduce mills to cents, *divide them by* 10.

3. In 235 cents, how many dollars? *Ans.* $2.35.

Reduce the following to the denominations indicated:

4. 563 cents to dollars.
5. 895 cents to dollars.
6. 1263 cents to dollars.
7. 5770 cents to dollars.
8. 268 mills to dollars.
9. 3275 mills to cents.

ADDITION OF U.S. MONEY.

1. George paid \$2.45 for a sled, and \$1.63 for a pair of skates: what was the cost of both?

ANALYSIS.—\$2 and \$1 are \$3; 45 cts. and 63 cts. are 108 cts., or \$1.08, which added to \$3, make \$4.08. Therefore, etc.

2. What is the sum of \$5.17 and \$12.30?

3. If a hat costs \$5.50, and a vest \$9.75, what is the cost of both?

4. A farmer sold a sheep for \$5.50, and a calf for \$7.30: what did he receive for both?

5. The price of a reader is 87 cts., and an arithmetic 63 cts.: what is the price of both?

6. If a man pays \$3.25 a day for board, and 85 cents for cigars, what are his daily expenses for both?

SLATE EXERCISES.

8. Upon what principle is U. S. Money founded?

It is founded upon the *Decimal Notation.*

9. How are its operations performed?

Its *operations* are the *same* as the *corresponding* operations in *whole numbers* and *Decimal Fractions.*

1. What is the sum of \$10.625; \$16.078; \$28?

ANALYSIS.—We write dollars under dollars, cents under cents, etc., and beginning at the right, add as in simple numbers, placing the *decimal point* in the amount under those in the numbers added. (P. 25, Q. 9.)

\$10.625
16.078
28.00
\$54.703

10. How add United States money?

Write dollars under dollars, cents under cents, etc., and add as in simple numbers, placing the decimal point in the amount under those in the numbers added.

NOTE.—If any of the given numbers have *no cents,* their place should be be supplied by *ciphers.*

	(2.)	(3.)	(4.)	(5.)
	$430.451	$641.375	$890.40	$2056.625
	205.06	80.06	708.00	140.50
	128.007	65.007	25.56	68.08
Ans.	763.518	240.25	7.07	9.315

6. What is the sum of $85.10; $164.07; and $35.20?

7. What is the sum of $207.56; $500.65; and $61.52?

8. Paid $8.75 for a barrel of flour; $5.25 for 2 barrels of apples; and $7 for a ton of coal: what was the amount of my bill?

9. A farmer bought a horse for $120.875; a yoke of oxen for $95; and a cart for $68.50: what did he pay for all?

10. A merchant sold goods amounting to $150.35 to one customer; to another $96.40; to another $110; and to another $200.68: what amount did he sell to all?

11. Add 120 dollars, 5 cents, and 3 mills; 45 dollars, and 7 mills; 78 cents, and 6 mills.

12. Add 7 dollars, and 7 cents; 10 dollars, and 5 mills; 217 dollars, and 45 cents; and 31 dollars.

13. Add 371 dollars, 40 cents, and 8 mills; 710 dollars; 90 dollars, and 35 cents; and 219 dollars.

14. Add 1000 dollars; 100 dollars, and 10 cents; 93 cents; 860 dollars, and 8 cents; 5 dollars and 95 cents.

15. What is the sum of 1500 dollars and 8 cents + 807 dollars, 60 cents, and 7 mills + 763 dollars, 3 cents and 5 mills + 85 cents and 8 mills?

16. A lady bought a dress for $45.63; a shawl for $87.625; a collar for $15.375; a pocket-handkerchief for $7.50; what was the amount of her bill?

17. Bought an overcoat for $35.75; a dress-coat for $28.62½; and a vest for $9.12½: required the amount.

SUBTRACTION OF U.S. MONEY.

1. William having $7.62, gave $2.50 for a cap: how much did he have left?

ANALYSIS.—$2 from $7 leave $5; and 50 cents from 62 cents leave 12 cents. Therefore, etc.

2. Henry gave a 10 dollar bill to pay for a hat, the price being $7.50: how much change should he receive?

3. The price of a grammar is 85 cts., and that of a geography $1.30: what is the difference in their prices?

4. A father earns $10.75 a week, and his son $8.50: how much more does the former earn than the latter?

5. A man paid $12.60 for a gal. of brandy, and $8.25 for a bar. of flour: required the difference in cost?

SLATE EXERCISES.

1. A person having $356.07, paid $109.625 for a horse: how much did he have left?

ANALYSIS.—We write the less number under the greater, dollars under dollars, cents under cents, etc. Subtract as in simple numbers, and place the *decimal point* in the remainder under that in the subtrahend. (P. 38, Q. 9.)

OPERATION.

$356.07
109.625
$246.445

11. How subtract United States money?

Write the less number under the greater, dollars under dollars, cents under cents, etc., and subtract as in simple numbers, placing the decimal point in the remainder under that in the subtrahend.

NOTE.—If either of the given numbers has no *cents*, their place should be supplied by ciphers.

	(2.)	(3.)	(4.)	(5.)
From	$65.875	$110.46	$68.004	$100.00
Take	46.29	95.375	19.086	0.875

6. George gave $1.75 for his geography, and $0.875 for his arithmetic: what was the difference in cost?

7. A lady bought articles amounting to $29.375, and gave the clerk a 50 dollar bill: how much change ought she to receive?

8. Bought a coat for $25.75; pants for $14; vest for $11.50; and sold wood to the tailor amounting to $50: how much am I indebted to him?

9. If you have $407 on deposit, and check out $219.625, how much will you have left in bank?

10. Find the difference between $117.45 and $201.03?

11. Find the difference between $1000 and 1000 cts.?

12. From two hundred dollars and seven cents, take forty dollars and 5 mills.

13. From one hundred dollars and 6 cents, take five dollars and 20 cents.

14. From $300, take 3 dol., 3 cts., and 3 mills.

15. A father gave one daughter a music-box worth $75.375, the other a sewing-machine worth $55.67: what was the difference in their cost?

MULTIPLICATION OF U. S. MONEY.

1. What will 3 chairs cost, at $7.50 each?

ANALYSIS.—3 chairs will cost 3 times as much as 1 chair. Now 3 times $7 are $21, and 3 times 50 cts. are 150 cts., equal to $1.50, which added to $21, make $22.50. Therefore, etc. (P. 52, Note.)

2. What cost 4 fruit-knives, at $2.12 apiece?

3. What cost 5 bouquets, at $3.50 apiece?

4. What cost 6 paper-folders, at 75 cents apiece?

5. At $4.10 a box, what will 8 boxes of lemons come to?

6. At $11.20 apiece, what will 10 dresses cost?

SLATE EXERCISES.

1. What will 18 ploughs cost, at $13.125 apiece?

ANALYSIS.—If 1 plough costs $13.125, 18 ploughs will cost 18 times as much. We multiply in the usual way, and from the right of the product point off *three* figures for cents and mills; because there are *three* places of cents and mills in both factors. (P. 139, Q. 28.)

$13.125
18
105000
13125
$236.250

12. How multiply United States money?

Multiply as in simple numbers, and on the right of the product, point off as many figures for cents and mills as there are decimal places in both factors.

NOTE.—In *United States Money*, as in simple numbers, the *multiplier* must be considered an *abstract* number.

2. If you spend 87½ cents a day, what will you spend in 7 days?

SOLUTION.—87½ cts.=$0.875, and $0.875 × 7=$6.125, *Ans.*

	(3.)	(4.)	(5.)	(6.)
Mult.	$39.35	$60.075	$100.008	$82650
By	11	.15	6.5	.75

7. What will 36 chickens come to, at 62½ cts. each?

8. If a man earns $9.50 a week, what will be his wages for 52 weeks?

9. At $1.37½ per yard, what will a dress containing 20.5 yards of grenadine come to?

10. At $7.50 a ton, what will 100.5 tons of coal cost?

11. What cost 18 pianos, at $750 apiece?

12. What is the amount of a man's expenses for 12 months, if he spends $86.50 a month?

13. What cost 25 building-lots, at $1250.50 a lot?

14. At \$4.50 each per day, what will be the hotel expenses of 6 persons for 4 weeks?

15. What is the sum of 12 times $87\frac{1}{2}$ cents, and 15 times $62\frac{1}{2}$ cents?

16. What is the difference between 20 times \$17.65, and 17 times \$25.40?

17. A farmer bought 12 calves, at \$7.60 each; and 20 sheep, at \$4.75 each: how much did he pay for both lots; and what is the difference in their cost?

DIVISION OF U. S. MONEY.

1. If 9 oranges cost 63 cents, what will 1 orange cost?

ANALYSIS.—If 9 oranges cost 63 cents, 1 orange will cost 1 ninth of 63 cts., which is 7 cts. Therefore, etc. (P. 63, Q. 10.)

2. If 7 sheep cost \$35, what will one cost?

3. If 8 yards of velvet cost \$72, what will 1 cost?

4. A man laid out \$50 in vests, which were \$5 apiece: how many did he buy?

5. How many hand-carts, at \$6, can be bought for \$300?

SLATE EXERCISES.

1. Sold 6 hats for \$42.75: what was that apiece?

OPERATION.

6)\$42.750

Ans. \$7.125

ANALYSIS.—1 hat is 1 sixth of 6 hats; hence 1 hat is worth 1 sixth of \$42.75, which is \$7.125.

The object of this example is to *divide* the sum of \$42.75 into 6 equal parts. (P. 63, Q. 10.) Dividing as in simple numbers, there is a remainder of 3 cents which we reduce to mills, and dividing as before, point off three figures for cents and mills.

13. How divide United States money by an abstract number?

Reduce the dividend to mills, and divide as in simple numbers. The quotient will be mills, which must be reduced to dollars and cents.

NOTE.—If there is a remainder, write the sign + after the quotient.

2. How many hats, at \$7.125, can be bought for \$42.75?

ANALYSIS.—The object here is to find *how many times* one sum of money is contained in another. But the divisor contains dollars, cents, and *mills*, while the dividend contains *dollars* and *cents* only. We therefore reduce the latter to mills, and then divide as in simple numbers. (P. 67, Q. 10.)	OPERATION. \$7.125)\$42.750(6 42.750

13, *a*. How divide U. S. Money by U. S. Money?

Reduce the divisor and dividend to the same denomination, and divide as in simple numbers. The quotient will be times, or an abstract number. (P. 63, Q. 10.)

NOTES.—1. In business matters it is rarely necessary to carry the quotient beyond mills.

2. If there is a remainder after all the figures are divided, *annex ciphers*, and continue the division as far as desirable, considering the ciphers annexed as decimals of the dividend.

(3.)	(4.)	(5.)	(6.)
7)\$92.694	8)\$114	9)\$12.791	6)\$0.804
\$13.242	\$14.25	\$1.421+	\$0.134

7. How many times are \$8 contained in \$90.47?

8. How many times are \$75 contained in \$900?

9. How many melons, at \$0.25 each, can be purchased for \$15.75?

10. How many pounds of butter, at \$0.30, can be had for \$25.65?

11. If I pay \$14.875 for 7 baskets of peaches, how much is that a basket?

12. A stationer sold 5 slates for \$0.625: what was that apiece?

13. At $20 apiece, how many yearlings can be bought for $280?

14. How many acres of land, at $12.50 per acre, can you buy for $1000?

15. Paid $43 for 8 excursion-tickets: how much was that for each ticket?

16. A clerk agreed to work 12 months for $427.56: what was that per month?

17. If $1600.75 are divided equally among 25 persons, how much will each receive?

18. Sold 280 sheep for $658: what was that per head?

19. Sold 35 doz. eggs for $8.75: what was that a doz.?

20. Paid $2675.75 for 278 tons of coal: what was the cost per ton?

QUESTIONS FOR REVIEW.

1. A man bought 6 cords of wood at $4.17, and 5 tons of coal at $7.375: what did he pay for both?

2. If you buy 10 pen-knives for $6.75, and sell them at 85 cents apiece, how much will you make or lose?

3. What is the difference between 7 times $8.50, and 9 times $7.625?

4. What is the difference between 11 times $17.65, and 8 times $19.48?

5. Bought 8 boxes of raisins at $6.40, and sold them at $9.63 a box: how much was made on them?

6. A traveler was robbed of $375, and had $159.60 left: how much money had he before the robbery?

7. If you buy 12 melons for $3.96, and sell them at 50 cents each, how much will you make on each?

8. A grocer bought 12 bags of coffee for $121.92, but finding it damaged sold it for $30.20 less than cost: for what did he sell it per bag?

9. Bought 111 barrels of flour for \$897.50: for how much must I sell it per barrel to make \$300?

10. Paid \$1162.50 for 25 acres of land: what is that per acre?

11. Paid \$6785 for 100 oxen: what was that apiece?

12. How many horses, at \$150, will \$10650 buy?

13. What is the sum and difference of \$567.625 and \$945.50?

14. How many tubs of butter, at \$16.50 each, can be bought for \$206.25?

15. Exchanged 75 barrels of apples worth \$150, for 25 barrels of cider: what did the cider cost per barrel?

16. How many pair of shoes, at \$1¼, can be had for 5000 pounds of rice, at 12½ cents a pound?

APPLICATIONS OF U. S. MONEY.

BILLS.

14. What is a bill?

A ***Bill*** is a *written statement* of goods sold, services rendered, etc., and should include the various items, the *price* of each, the *date*, and the *place* of the transaction.

15. How are bills receipted?

A ***Bill is receipted*** when it contains the words, "Received payment," and is signed by the person to whom it *is due*, or his agent.

16. What is the meaning of the terms *Debtor* and *Creditor*?

A ***debtor*** is a person who *owes* a debt.

A ***creditor*** is one to whom a debt is *owed*.

NOTES.—1. The abbreviation DR. denotes *debit* or *debtor*; CR., *credit* or *creditor*; *per* signifies *by*, and the character @, *at*.

2. To familiarize the learner with the *form* of bills, the manner of receipting them, etc., it is advisable for him to copy the following, in a neat hand, upon his slate or paper.

Find the amount of the following bills:

(1.) NEW YORK, June 3d, 1871.

HON. GEORGE PEABODY,

Bought of HORACE WEBSTER.

7	lbs. coffee,	@	$0.38	. . . . .	$2.66
12	" sugar,	"	.16	. . . . .	1.92
6	" corn starch,	"	.11	. . . . .	.66
5	" tea,	"	.87	. . . . .	4.35

Amount

Received Payment,
HORACE WEBSTER.

(2.) MOBILE, Feb. 21st, 1871.

GEORGE WALKER, ESQ.,

To DANIEL KINGSBURY & CO, *Dr.*

To	15 yds. silk,	@	$2.35	
"	11 " muslin,	"	.29	
"	12 pair hose,	"	.42	
"	12 " gloves,	"	1.50	
"	6 parasols,	"	3.50	

Amt.

Rec'd Pay't by Note,
D. KINGSBURY & CO.,
By S. BARRET.

(3.) CHICAGO, May 21st, 1871.

JOHN MURDOCK, ESQ., *in acct. with* DAVID JOY & CO.
Dr.

For	12 pair shoes,	@	$1.62		
"	6 " thick boots,	"	2.75		
"	10 " slippers,	"	.88		
"	24 " woollen hose,	"	.30		
	Carried forward,				$51.94

	Brought forward,			$51.94
	Credit.			
By	6 bushels wheat,	@	$1.50	
"	14 " oats,	"	.60	
"	3 barrels cider,	"	2.75	
"	10 barrels potatoes,	"	1.87	
What is the balance?				44.35
				Balance, $7.59

(4.)

MESSRS. J. H. BURTIS & CO.,

Bought of SCHERMERHORN & WILSON.

12 slates @ $.13; 3 blackboards @ $9.50; 6 boxes of crayons @ $.68; 36 inkstands @ $.12; 2 small globes @ $5.25. Required the amount.

(5.)

JAMES BARBER *in acct. with* W. C. YOUNG.

Dr.

For 10 shovels @ $1.67; 12 hoes @ 1.25; 6 rakes @ $1.50; 4 axes @ $2.63.

Credit.

By 12 days work, man, @ $2.00; 10 days work, self and team, @ $3.60; 20 cords wood @ $3.10; 15 shade-trees @ $1.75.

What is the balance due on the above?

6. George Morris of Chicago bought of A. T. Stewart & Co. 18 yds. of silk, at $2.63; 14 yds. Empress cloth, at $1.75; 12 yds. of poplin, at $2.375; 15 yds. of French lawn, at $0.65; 6 pair of gloves, at $1.85; 6 pocket-handkerchiefs, at $1.25; 3 parasols, at $3.60. Required the amount.

COMPOUND NUMBERS.

1. **What are *Simple* Numbers?**

Simple Numbers are those which contain units of *one denomination* only; as, two, four, 3 apples, 4 quarts, etc.

2. **What are *Compound* Numbers?**

Compound Numbers are those which contain *two* or *more denominations* of the *same nature;* as, 4 bushels and 3 pecks; 3 days and 5 hours.

ILLUSTRATION.—Suppose, for example, we apply the *inch* as a unit of measure to the side of a table, and find it equal to 30 such measures. Again, if we employ the *foot* (12 in.) as the unit, it is equal to two such measures, and 6 in. over. Now as 6 inches = ½ foot, we may call its length 2½ feet, or 2 feet, 6 inches. The *former* expression contains units of but *one* denomination, viz., *feet;* therefore, it is a *simple* number. The latter contains units of *two different* denominations, which are of the same nature, viz., *feet* and *inches;* therefore it is a *compound* number.

But the expression 2 *feet* and 4 *pounds* is not a compound number; for the units are of *unlike* natures.

NOTE.—Compound numbers are often called *Denominate Numbers.*

3. **When it is said that a cane is 3 feet long, what is the kind of number used?**

A ***Simple Number;*** because it contains but *one* denomination, viz., feet.

4. **If we say that a cane is 2 feet and 10 inches long, what kind is the number?**

A ***Compound Number;*** because it contains *two* denominations of the same nature, viz., feet and inches.

What kind of a number is 6 days? Why?

What kind is 5 pounds 2 shillings and 6 pence? Why?

What kind is each of the following: Two? Three? 12 oranges? 7 pounds and 5 ounces? 10 dollars and 25 cents? 10 oxen?

UNITED STATES MONEY.

5. What is United States Money?

United States Money is the national currency of the United States, and is often called *Federal Money.*

6. What are its denominations?

Eagles, dollars, dimes, cents, and *mills.*

TABLE.

10 mills (*m.*)	are	1 cent,	*ct.*
10 cents	"	1 dime,	*d.*
10 d., or 100 cts.	"	1 dollar, $, or	*dol.*
10 dollars	"	1 eagle,	*E.*

7. Of how many kinds is U. S. money?

Two, *Paper* and *Metallic.*

8. What is the *paper* money of the U. S.?

The ***Paper Money*** of the U.S. consists of *Treasury-notes* issued by the Government, known as Greenbacks, and *Bank-notes* issued by Banks.

NOTE.—*Paper money* is called *Paper Currency.*

Treasury-notes less than $1 are called *Fractional Currency.*

9. What is *metallic* money?

Metallic Money consists of *stamped pieces* of metal, called *coins.* It is also called *specie,* or *specie currency.*

NOTE.—For exercises in U. S. money see pp. 143–157.

10. Of what do the coins of the United States consist?

Gold coins, *silver* coins, and the *minor* coins.

11. Name the coins of each.

The *gold* coins are the *double-eagle, eagle, half-eagle, quarter-eagle, three-dollar,* and *dollar* piece.

The *silver* coins are the *trade dollar, half-dollar, quarter-dollar, twenty-cent piece,* and *dime.*

The *minor* coins are the nickel *5-cent* and *3-cent* pieces, and the bronze-*cent.*

ENGLISH MONEY.

12. What is *English* money?

English Money is the national currency of Great Britain, and is often called *Sterling Money.*

13. What are the denominations?

Pounds, shillings, pence, and *farthings.*

TABLE.

4 farthings (*qr.* or *far.*)	are	1 penny,	*d.*
12 pence	"	1 shilling,	s.
20 shillings	"	1 pound or sovereign,	£.
21 shillings	"	1 guinea,	*g.*

NOTES.—1. The legal value of a *pound Sterling*, or *sovereign*, is $4.8665; the value of an English *shilling* is 24⅓ cents; and that of a *penny* about 2 cents.

2. Farthings are commonly expressed as fractions of a penny. Thus, 1 far.=¼d.; 2 far.=½d.; 3 far.=¾d.

1. How many farthings in 5 pence?

ANALYSIS.—Since there are 4 farthings in a penny, there must be 4 times as many farthings as pence, and 4 times 5 are 20 pence. Therefore, etc.

2. How many pence in 3 shillings? In 5 s.? In 7 s.?

3. How many shillings in £3? In £5? In £10?

4. In 15 farthings how many pence?

ANALYSIS.—Since in 4 farthings there is 1 penny, in 15 far. there are as many pence as 4 far. are contained times in 15 far; and 4 is in 15, 3 times and 3 over. Therefore, in 15 far. there are 3d. and 3 far. over, or 3¾d.

5. How many shillings in 18 pence? In 24d.?

6. How many pounds in 21 shillings? In 30 s.? In 40 s.? In 85 s.? In 100 s.?

*** If the teacher desires further practice upon the Tables, as they are recited, he will find corresponding Examples in the Slate Exercises, pp. 173–178.

TROY WEIGHT.

14. For what is Troy Weight used?

For weighing *gold, silver*, and *jewels.*

15. What are the denominations?

Pounds, ounces, pennyweights, and grains.

TABLE.

24 grains (*gr.*)	are	1 pennyweight,	*pwt.*
20 pennyweights	"	1 ounce,	*oz.*
12 ounces	"	1 pound,	*lb.*

NOTE.—The best method of imparting to children a correct idea of *Weights* and *Measures*, is to let them *see* and *handle* the actual standards, or some material objects which are equal to the several *units* of *length, surface, capacity,* and *weight.* In this way, the Compound Tables afford a wide field for *object teaching.*

1. How many grains in 2 pennyweights?
2. How many pennyweights in 3 ounces? In 5 oz.?
3. How many ounces in 3 pounds? In 5 pounds?
4. How many ounces in 40 pwt.? In 45 pwt.?

AVOIRDUPOIS WEIGHT.

16. For what is Avoirdupois weight used?

For weighing all *coarse articles;* as, hay, cotton, groceries, etc., and all metals except *gold* and *silver.*

17. What are the denominations?

Tons, hundreds, pounds, and *ounces.*

TABLE.

16 ounces (*oz.*)	are	1 pound,	*lb.*
100 pounds	"	1 hundred weight,	*cwt.*
20 cwt., or 2000 lbs.,	"	1 ton,	*T.*

8 oz. = $\frac{1}{2}$ lb.; 4 oz. = $\frac{1}{4}$ pound.

NOTES.—1. The *ounce* is often divided into *halves*, *quarters*, etc.

2. In business transactions, the *dram*, the *quarter* of 25 lbs., and the *firkin* of 56 lbs., are not used as *units* of Avoirdupois Weight.

3. *Net* weight is the weight of goods, without the *bag*, *cask*, etc.

4. *Gross* weight is the weight of goods with the bag, cask, or box in which they are contained. It calls 28 lbs. a *quarter*, 112 pounds a *hundred* weight, and 2240 pounds a *long ton*.

1. How many ounces in 3 pounds? In 4 lbs.?
2. How many pounds in 3 quarters? In 5 qrs.?
3. How many hundreds in 3 tons? In 5 tons?
4. In 40 ounces, how many pounds? In 48 oz.?
5. In 30 hundreds, how many tons? In 60 cwt.?
6. In 3500 pounds, how many tons? In 5000 lbs.?

APOTHECARIES' WEIGHT.

18. For what is Apothecaries' Weight used?

For *mixing medicines*.

19. What are the denominations?

Pounds, *ounces*, *drams*, *scruples*, and *grains*.

TABLE.

20 grains (*gr.*)	are	1 scruple,	*sc.*,	or ℈.
3 scruples	"	1 dram,	*dr.*,	or ʒ.
8 drams	"	1 ounce,	*oz.*,	or ℥.
12 ounces	"	1 pound,		℔.

NOTE.—The only difference between *Troy* and *Apothecaries'* weight, is in the *division* of the *ounce*. The *pound*, *ounce*, and *grain* are the same in each.

1. How many grains in 2 scruples? In 3 scruples?
2. How many scruples in 4 drams? In 8 drams?
3. How many drams in 5 ounces? In 100 ounces?
4. How many ounces in 6 pounds? In 12 pounds?

LINEAR MEASURE.

20. For what is Linear Measure used?

For measuring that which has *length*, without *breadth*; as, lines, distances, etc. It is often called *Long Measure*.

21. What are the denominations?

Leagues, miles, furlongs, rods, yards, feet, and *inches*.

TABLE.

12 inches (*in.*)	are	1 foot,	*ft.*
3 feet	"	1 yard,	*yd.*
$5\frac{1}{2}$ yds., or $16\frac{1}{2}$ ft.	"	1 rod, perch, or pole,	*r.*, or *p.*
40 rods,	"	1 furlong,	*fur.*
8 fur., or 320 rods	"	1 mile	*m.*
3 miles	"	1 league,	*l.*

NOTE.—The inch is commonly divided into *halves, fourths, eighths*, or *tenths*; sometimes into *twelfths*, called *lines*.

1. Draw a straight line 2 inches long on your slate or blackboard.

2. Draw one 4 in. long. 6 in. long. 9 in. long. A foot long. A yard long.

3. How long is your pencil? This pen-knife? This pen-holder? This paper-folder? This ruler?

4. How long is this table? How wide? How long is the school-room? How wide? How high? How long is the play-ground? *

5. How many inches in 3 feet? In 5 feet? In 8 feet?

6. How many inches in 2 ft. and 5 in.? 4 ft. and 6 in.?

7. How many feet in 4 yards? In 7 yds.? In 9 yds.?

8. How many feet in 5 yards and 2 feet? In 6 yds. and 4 ft.?

* This exercise should be varied, and continued till the class obtain a clear idea of the ordinary measures of length.

9. How many miles in 5 leagues? In 8 leagues?

10. How many feet in 2 rods? In 3 rods?

11. In 37 inches, how many feet? In 60 in.? In 75 in.? In 100 in.?

12. In 18 feet, how many yards? In 28 ft.? In 40 ft.?

13. In 32 furlongs, how many miles? In 41 fur.? In 50 fur.?

CLOTH MEASURE.

22. **For what is Cloth Measure used?**

For measuring those articles of commerce whose *length only* is considered; as, cloths, laces, ribbons, etc.

23. **What are the denominations?**

The ***Linear Yard*** is the principal unit. This is divided into *quarters, eighths,* and *sixteenths.*

TABLE.

3 ft. or 36 in.,	are	1 yard, - - -	*yd.*
18 in.,	"	1 half yard, - -	$\frac{1}{2}$ *yd.*
9 in.,	"	1 quarter yard, -	$\frac{1}{4}$ *yd.*
$4\frac{1}{2}$ in.,	"	1 eighth " -	$\frac{1}{8}$ *yd.*
$2\frac{1}{4}$ in.,	"	1 sixteenth, " -	$\frac{1}{16}$ *yd.*

NOTE.—*Ells* Flemish, English, and French, are no longer used in the United States; and the *nail*, as a measure, is practically obsolete.

1. How many quarters in 14 in.? In 26 in.?

2. How many fourths in $3\frac{1}{4}$ yards? In $5\frac{3}{4}$ yards?

3. How many eighths in 25 in.? In 37 in.?

4. How many eighths in $2\frac{3}{8}$ yards? In $3\frac{5}{8}$ yards?

5. How many yards in 14 half yards? In 30 halves? In 35 halves?

6. How many yards in 25 fourths of a yard? In 32 fourths? In 48 sixteenths?

SQUARE MEASURE.

24. For what is Square Measure used?

For measuring *surfaces*, or that which has *length* and *breadth*, without *thickness;* as, land, flooring, etc. It is often called *Land Measure.*

25. What are the denominations?

Acres, square rods, square yards, square feet, and *square inches.*

TABLE.

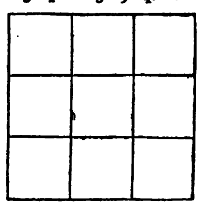
144	square in. (*sq. in.*)	are	1 square foot,	*sq. ft.*
9	square feet	"	1 square yard,	*sq. yd.*
$30\frac{1}{4}$ / $272\frac{1}{4}$	square yards, or / square feet,	"	1 sq. rod, perch, or pole,	*sq. r.*
160	sq. rods	"	1 acre,	*A.*
640	acres	"	1 square mile,	*M.*

NOTE.—The *acre* was formerly divided into 4 roods; but in practice the *rood* is no longer used as a *unit* of measure.

26. What is a Square?

A ***Square*** is a rectilinear figure which has *four* equal sides and *four right angles.* Thus,

3 sq. ft.×3=9 sq. ft.



9 sq. ft.=1 sq. yd.

A ***Square Inch*** is a square, each side of which is 1 inch in length.

A ***Square Yard*** is a square, each side of which is 1 yard in length.

NOTE.—The *corners* of any square figure, also of a table, a room, etc., are *right angles.*

1. Make a right angle upon your slate, or the blackboard.

2. Make a square inch.
3. Make a square whose side is 3 inches. 6 inches.
4. Make a square foot.
5. Make a square yard.
6. Divide a square yard into square feet.
7. Divide a square foot into square inches.
8. How many square inches in 2 sq. ft.? In 3 sq. ft.?
9. How many square feet in 3 sq. yds.? In 5 sq. yds.?
10. In 27 sq. feet, how many sq. yards? In 36 sq. feet?
11. What is the difference between 3 feet square, and 3 square feet?

CUBIC MEASURE.

27. **For what is Cubic Measure used?**

For measuring *solids;* as, timber, boxes of goods, the capacity of rooms, ships, etc. It is often called *Solid Measure.*

28. **What are the denominations?**

Cords, cubic yards, cubic feet, and *cubic inches.*

TABLE.

1728 cubic inches (*cu. in.*)	are	1 cubic foot,	*cu. ft.*
27 cubic feet	"	1 cubic yard,	*cu. yd.*
128 cubic feet	"	1 cord,	*C.*

28, ***a.*** **Describe a cord? A foot of wood?**

A ***Cord*** of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high: for, $8 \times 4 \times 4 = 128$.

A ***Cord Foot*** is *one* foot in length of such a pile; hence, 8 cord feet make one cord.

NOTE.—Timber is now measured by cubic feet and inches.

The old cubic ton of 40 feet of round timber, and 50 feet of hewn timber, has fallen into disuse in the United States.

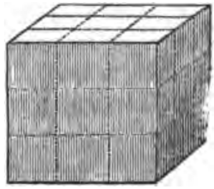
29. **What is a Cube?**

A *Cube* is a regular solid bounded by *six equal squares*, called its faces. Thus,

A ***Cubic Inch*** is a cube, each side of which is a square inch.

A ***Cubic Yard*** is a cube, each side of which is a square yard.

27 cu. ft. = 1 cu. yd.



1. Draw a cubic inch.
2. Draw a cube whose sides are 3 inches square.
3. Draw a cubic foot.
4. How long and wide must a block of marble be, whose height is 3 feet, to form a cubic yard?
5. How many cubic feet in 2 cubic yards?
6. How many cubic yards in 54 cubic feet?
7. In 2 cords, how many cubic feet?

DRY MEASURE.

30. **For what is Dry Measure used?**

For measuring *grain, fruit, salt,* etc.

31. **What are the denominations?**

Chaldrons, bushels, pecks, quarts, and *pints.*

TABLE.

2 pints (*pt.*),	are 1 quart,	*qt.*
8 quarts	" 1 peck,	*pk.*
4 pecks, or 32 qts.,	" 1 bushel,	*bu.*
36 bushels	" 1 chaldron,	*ch.*

Notes.—1. The *dry* quart is equal to $1\frac{1}{6}$ liquid quart nearly.

2. The *chaldron* is used for measuring coke and bituminous coal.

1. In 8 pints, how many quarts? In 16 pints?
2. In 32 quarts, how many pecks? In 40 quarts?
3. How many pecks in 5 bushels? In 7 bushels?
4. How many quarts in 5 pecks? In 9 pecks?
5. How many bushels in 12 pecks? In 17 pecks?
6. How many quarts in 2 bushels and 3 pecks?
7. How many quarts in 3 pecks and 4 quarts?
8. How many bushels in 40 quarts? In 64 quarts?

LIQUID MEASURE.

32. For what is Liquid Measure used?

For measuring *milk, wine, vinegar, molasses*, etc.

33. What are the denominations?

Hogsheads, barrels, gallons, quarts, pints, and *gills*.

TABLE.

4	gills (*gi.*)	are	1 pint,	*pt.*
2	pints	"	1 quart,	*qt.*
4	quarts	"	1 gallon,	*gal.*
$31\frac{1}{2}$	gallons	"	1 barrel, *bar.*, or	*bbl.*
63	gallons	"	1 hogshead,	*hhd.*

NOTES.—1. *Liquid Measure* is often called *Wine Measure*.
2. The old *Beer Measure* is practically obsolete in this country

1. How many gills in 4 pints? In 10 pints?
2. How many pints in 7 quarts? In 9 quarts?
3. How many quarts in 5 gallons? In 10 gallons?
4. In 20 quarts, how many gallons?
5. In 24 pints, how many quarts?
6. In 16 gills, how many pints?
7. In 24 gills, how many pints? How many quarts?
8. In 32 pints, how many quarts? How many gallons?
9. How many gallons in 2 barrels?
10. How many gallons in 2 hogsheads?

CIRCULAR MEASURE.

34. **For what is Circular Measure used?**

For measuring *angles, land, latitude* and *longitude, the motion of the heavenly bodies*, etc.

35. **What are the denominations?**

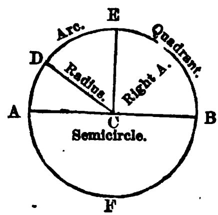
Signs, degrees, minutes, and *seconds.*

TABLE.

60 seconds (″)	are	1 minute,	′
60 minutes	"	1 degree,	°
30 degrees	"	1 sign,	*s.*
12 signs, or 360°	"	1 circumference,	*cir.*

NOTE.—*Signs* as a measure are used only in Astronomy.

36. **What is a Circle?**



A ***Circle*** is a plane figure bounded by a curve line, every part of which is *equally distant* from a point within called the *center.*

37. **What is the Circumference of a Circle?**

The ***Circumference of a Circle*** is the curve line by which it is bounded. It is divided into 360°.

38. **What is the Diameter?**

The ***Diameter*** is a *straight line* drawn through the *centre,* terminating at each end in the *circumference.*

39. **What is the Radius?**

The ***Radius*** is a straight line drawn from the *center* to the *circumference,* and is equal to *half* the diameter.

40. **What is an Arc of a Circle?**

An ***Arc of a Circle*** is any part of the circumference.

In the preceding figure, A D E B F is the circumference; A B the diameter; C A, C D, C E, etc., are radii; A D, D E, etc., are arcs.

Draw a circle. Draw a diameter. Draw another diameter perpendicular to the first.

NOTE.—These two diameters divide the circumference into four equal parts, called *quadrants*.

41. **How many degrees in a quadrant?**

Ninety.

42. **How many and what angles do these two diameters form?**

Four right angles.

43. **How many degrees in a right angle?**

Ninety.

MEASUREMENT OF TIME.

44. **What are the denominations of Time?**

Centuries, years, months, weeks, days, hours, minutes, and seconds.

TABLE.

60 seconds (*sec.*)	are	1 minute,	*min.*
60 minutes	"	1 hour,	*h.*
24 hours	"	1 day,	*d.*
7 days	"	1 week,	*w.*
365 days, or 52 w. and 1 d.,	"	1 common year,	*c. y.*
366 days	"	1 leap year,	*l. y.*
12 calendar months (*mo.*)	"	1 civil year,	*y.*
100 years	"	1 century,	*cen.*

NOTE.—In most business transactions, 30 days are considered a *month*. Four weeks are sometimes called a *lunar month*.

45. **What is a common year?**

A ***common year*** is one which contains 365 days.

46. **What is a solar year?**

A ***solar year*** is the time in which the earth revolves around the sun, and equals 365 d. 5h. 48 min. and 49.7 sec.

NOTE.—The *excess* of the *solar* above the *common* year is about 6 hours, or $\frac{1}{4}$ of a day, nearly; hence, in 4 years, it amounts to about 1 day.

47. **What is a leap year?**

A ***leap year*** is one which contains 366 days.

48. **How caused, and why so called?**

It is caused by the *excess* of a *solar* above a *common* year; and is so called because it *leaps over* the limit, or runs on one day more than a common year.

This day is given to *February*, because it is the shortest month; hence, in leap years, February has 29 days.

49. **What is a civil year?**

A ***civil year*** is the year adopted by government for computing time, and includes both *common* and *leap* years as they occur.

50. **How is the civil year divided?**

It is divided into 12 *calendar months*, viz.:

January	(*Jan.*),	the	first month,	has	31	days.
February	(*Feb.*),	"	second	"	28	"
March	(*Mar.*),	"	third	"	31	"
April	(*Apr.*),	"	fourth	"	30	"
May	(*May*),	"	fifth	"	31	"
June	(*June*),	"	sixth	"	30	"
July	(*July*),	"	seventh	"	31	"
August	(*Aug.*),	"	eighth	"	31	"
September	(*Sept.*),	"	ninth	"	30	"
October	(*Oct.*),	"	tenth	"	31	"
November	(*Nov.*),	"	eleventh	"	30	"
December	(*Dec.*),	"	twelfth	"	31	"

NOTE.—The following couplet will aid the learner in remembering the months that have 30 days each:

"Thirty days hath September,
April, June, and November."

Each of the other months has 31 days, except February, which in common years has 28 days, but in leap years, 29.

51. Into how many seasons is the year divided?

Four, viz.: *Spring*, *Summer*, *Autumn*, and *Winter*.

52. Name the months of each season?

Spring	consists of	March, April, and May.
Summer	"	June, July, and August.
Autumn	"	September, October, and November.
Winter	"	December, January, and February.

MISCELLANEOUS TABLES.

12 things	are	1 dozen.		12 gross	are	1 great gross.
12 doz.	"	1 gross.		20 things	"	1 score.

24 sheets	are	1 quire of paper.	2 reams	are	1 bundle.
20 quires	"	1 ream "	5 bundles	"	1 bale.

2 leaves	are	1 folio.
4 leaves	"	1 quarto, or 4to.
8 leaves	"	1 octavo, or 8vo.
12 leaves	"	1 duodecimo, or 12 mo.
18 leaves	"	1 eighteen mo.
24 leaves	"	1 twenty-four mo.

NOTE.—The terms *folio, quarto, octavo*, etc., denote the number of leaves into which a sheet of paper is folded in making books.

Aliquot Parts of a Dollar, or 100 Cents.

50 cents = $\$\frac{1}{2}$.	$12\frac{1}{2}$ cents = $\$\frac{1}{8}$.
$33\frac{1}{3}$ cents = $\$\frac{1}{3}$.	10 cents = $\$\frac{1}{10}$.
25 cents = $\$\frac{1}{4}$.	$8\frac{1}{3}$ cents = $\$\frac{1}{12}$.
20 cents = $\$\frac{1}{5}$.	$6\frac{1}{4}$ cents = $\$\frac{1}{16}$.
$16\frac{2}{3}$ cents = $\$\frac{1}{6}$.	5 cents = $\$\frac{1}{20}$.

REDUCTION.

1. What is Reduction?

Reduction is *changing a number* from one denomination to another, without *altering* its *value*. It is of two kinds, *Ascending* and *Descending*.

2. What is Reduction Descending?

Reduction Descending is changing *higher* denominations to *lower;* as, feet to inches, etc.

3. What is Reduction Ascending?

Reduction Ascending is changing *lower* denominations to *higher;* as, inches to feet, etc.

To Reduce Higher Denominations to Lower.

1. How many farthings are there in £16, 5s. 4d. 2 far.?

OPERATION.

£16, 5s. 4d. 2 far.
20
325s.
12
3904d.
4
15618 far. *Ans.*

ANALYSIS.—Since there are 20s. in a pound, there must be 20 times as many shillings as pounds, *plus* the given shillings. Now 20 times 16 are 320s., and 320s. + 5s. = 325s. Again, since there are 12d. in a shilling, there must be 12 times as many pence as shillings, plus the given pence. Now 12 times 325 are 3900, and 3900d. + 4d. = 3904d. Finally, since there are 4 far. in a penny, there are 4 times as many farthings as pence, plus the given farthings. Now 4 times 3904 are 15616 far., and 15616 far. + 2 far. = 15618 far. Therefore, etc.

4. How reduce *higher* denominations to *lower?*

Multiply the highest denomination by the number required of the next lower to make one of the higher, and to the product add the lower denomination.

Proceed in this manner with the successive denominations, till the one required is reached.

2. Reduce £5, 6s. $9\frac{1}{2}$d. to farthings?

SUGGESTION.—£5, 6s. $9\frac{1}{2}$d. = £5, 6s. 9d. 2 far. *Ans.* 5126 far.

3. Reduce £9, 1s. $6\frac{1}{4}$d. to farthings?

4. In 17s. 4d. 3 far., how many farthings?

5. In £43, 4s. how many pence?

6. In £115, how many farthings?

To Reduce Lower Denominations to Higher.

7. Reduce 15618 farthings to pounds?

ANALYSIS.—Since in 4 farthings there is 1 penny, in 15618 far. there are as many pence as 4 is contained times in 15618; and 15618 divided by 4=3904d. and 2 far. over. Again, since in 12d. there is 1s., in 3904d. there are as many shillings as 12 is contained times in 3904; and 3904÷12=325s. and 4d. over. Finally, in 325s. there are as many pounds as 20 is contained times in 325; and 325÷20=£16 and 5s. over. Therefore, etc.

OPERATION.

4) 15618 far.
12) 3904d. 2 far.
20) 325s. 4d.
£16, 5s.

Ans. £16, 5s. 4d. 2f.

5. How reduce *lower* denominations to *higher*?

Divide the given denomination by the number required of this denomination to make a unit of the next higher.

Proceed in this manner with the successive denominations, till the one required is reached. The last quotient, with the several remainders annexed, will be the answer.

NOTE.—The *remainders*, it should be observed, are the same denomination as the respective *dividends* from which they arise (P. 62, Q. 5, Rem.)

PROOF.—Reduction *Ascending* and *Descending prove* each other; for one is the *reverse* of the other.

8. Reduce 1231 pence to pounds? *Ans.* £5, 2s. 7d.

PROOF.—Reversing the operation we have 20 times 5 = 100s., and 100s. + 2s. = 102s. Again, 12 times 102 = 1224d., and 1224d. + 7d. = 1231d., the same as the given number of pence. Therefore, the work is right.

£5, 2s. 7d.
20
102s.
12
1231d.

9. In 1461 pence, how many pounds, shillings, and pence?

10. In 27035 farthings, how many pounds, etc.?

11. What will 25 pen-knives cost, at 2s. 6d. apiece?

SOLUTION—2s. 6d.=30d. Now 30d. × 25=750d., and 750d.= £3, 2s. 6d.

12. What will 75 slates cost, at 11 pence each?

13. How many pennyweights in 7 lb. 5 oz. troy?

14. How many grains in 10 lb. 7 oz. 6 pwt. 9 gr.?

15. Reduce 1561 pwt. to pounds and ounces?

16. Reduce 6575 grains to pounds, etc.

17. A goldsmith made 12 gold rings, each weighing 3 pwt. 4 gr.: how many ounces of gold did he use?

18. A lady bought a gold chain weighing 2 oz. 12 pwt., at $1.50 a pennyweight: how much did she pay for her chain?

19. Reduce 2265 ounces Avoirdupois to hundreds.

20. In 15 T. 2 cwt. 31 lb. 8 oz., how many ounces?

21. Reduce 100 tons, 75 lb. 4 oz. to ounces.

22. What will 5 lbs. 4 oz. of candy come to, at 6 cts. an ounce?

23. A farmer sold 2 tons, 375 lbs. of maple sugar, at 15 cts. a pound: how much did he receive for it?

24. In 5 lb., apothecaries' weight, how many drams?

25. In 7 lb. 4 oz., how many scruples?

26. In 469 scruples, how many apothecaries' pounds?

27. In 1578 grains, how many apothecaries' ounces?

28. How many feet in 45 rods?

NOTE.—For multiplying by $5\frac{1}{2}$ or $16\frac{1}{2}$, the number of yards or feet in a rod, see Note, p. 113.

29. How many feet in 12 miles, 10 rods, and 7 feet?

30. How many yards in 26 miles, 3 fur. 2 yards?

31. In 456 yards, how many rods?

REMARK.—To divide by $5\frac{1}{2}$ or $16\frac{1}{2}$ (the number of feet or yards in a rod) we reduce both the divisor and dividend to *halves;* then divide in the usual way. Thus, $5\frac{1}{2}$=11 halves, and 456=912 halves; now 11 is contained in 912, 82 times and 10 remainder. But the dividend is half yards; therefore the 10 remainder is half yards, and is equal to 5 yards. (P. 120, Q. 45.)

OPERATION.

$$\begin{array}{r|l} 5\frac{1}{2} & 456 \text{ yd.} \\ 2 & 2 \\ \hline 11 & 912 \\ \hline \end{array}$$

Ans. 82 r. 5 yd.

32. In 1560 feet, how many rods?

33. How many miles, rods, etc., in 11278 feet?

34. In 5 l. 17 m. 3 fur. 6 r. 10 ft., how many feet?

35. When the fare is 5 cts. a mile, what will it cost you to ride 15 leagues?

36. How many rods of fence are required on both sides of a road 2 miles long?

37. Allowing a military step to be $2\frac{1}{2}$ ft., how many steps will a soldier take in marching 5 miles?

38. How many quarters of a yard in 45 inches?

39. How many eighths of a yard in 27 inches?

40. Reduce 151 yards to sixteenths.

41. In 951 eighths of a yard, how many yards?

42. If you pay 8 cts. for $\frac{1}{2}$ yard of muslin, how much would you have to pay for 20 yards?

43. A lady paid $3 for $\frac{1}{4}$ yard of lace; what would a piece of 35 yards come to at that rate?

44. How many square feet in 160 sq. rods?

45. How many sq. feet in 5 A. 61 sq. rods?

46. How many sq. yd. in 21 A. 36 sq. rods?

47. How many sq. rods in a sq. mile?

48. In 851 sq. rods, how many acres?

49. In 75625 sq. yards, how many acres?

50. In 46273 sq. inches, how many sq. yards?

51. What will be the cost of a village-lot containing 20 sq. rods of land, at 25 cts. per sq. foot?

52. What will it cost to sod a park of 2 acres, at 12 cts. a sq. yard?

53. How many cu. in. in 41 cu. yards, 16 cu. feet?

54. How many cu. yards in 96365 cu. inches?

55. Reduce 4250 cu. feet to cords?

56. Reduce $75\frac{1}{2}$ cords to cu. feet?

57. Reduce 15 cords and 23 cu. feet to cu. feet?

58. At 4 cts. a cu. foot, what will it cost to excavate a cellar containing 450 cu. yards?

59. What will 12 cords of wood come to, at 5 cts. a cubic foot?

60. What is the worth of 168 cord feet of wood, at $4 a cord?

61. Reduce 6 bu. 3 pk. 5 qt. to pints?

62. How many bushels in 1647 quarts?

63. What cost 5 bushels of chestnuts, at 12 cts. a quart?

64. A lad bought 2 bushels of apples for $2.50, and sold them at 40 cts. a half peck: what was his profit?

65. How many quart boxes are required to hold 4 bu. 1 pk. of blackberries?

66. At 6 cts. a quart, how many bushels of peanuts can be bought for $6.42?

67. How many gills in 6 gal. 1 qt. 1 pt. 3 gills?

68. How many quarts in 3 hhd. 3 gal. 2 qt.?

69. In 1832 gills, how many gallons?

70. In 2560 quarts, how many hogsheads?

71. A milkman having a 15 gallon can full of milk, sold 15 quarts, and spilt the rest: how many quarts did he lose?

72. What cost a hogshead of maple syrup, at 25 cents a quart?

73. A druggist paid \$126 for a cask of alcohol containing 42 gal., and sold it at 20 cts. a gill: how much did he make?

74. How many seconds in 3 days, 5 hr. 17 minutes?

75. How many days in 4565 minutes?

76. How many minutes in 7 weeks, 5 days?

77. How many minutes in a common year?

78. How many common years in 48256 hours?

79. If a clock ticks seconds, how many times does it tick in a week?

80. At \$3.50 per day, what will it cost me to board 12 weeks?

81. If a man's pulse beats 73 times a minute, how many times will it beat in 31 days?

82. If a steamer sails 11 miles an hour, how long will it take her to sail 3000 miles?

83. Reduce 45° 13′ to seconds.

84. How many degrees in 10000″?

85. How many signs in 8275′?

86. The earth revolves 360° on its axis in 24 hours: how many degrees does it revolve in 1 hour? How far in 4 minutes?

87. How many sheets of paper in 5 reams, 10 quires?

88. How many reams in 12258 sheets?

89. If you pay \$2.50 a ream for paper, what is that a sheet?

90. How many crayons are there in 40 boxes, each containing a gross?

91. What will 25 gross of lead pencils cost, at 42 cts. a dozen?

92. Pens are packed in boxes containing a gross: how many pens are there in 6 boxes?

93. A man having 100 dozen eggs, sent them to market in 16 boxes: how many eggs did he put in a box?

MEASUREMENT OF RECTANGULAR SURFACES.

6. What is a Rectangular Figure?

A ***Rectangular Figure*** is one which has *four sides* and *four right angles.* (See next Fig.)

When *all* the sides are equal, it is called a *square*, when the *opposite* sides only are equal, it is called an *oblong*, or *parallelogram.*

7. What is the *area* of a figure?

The ***Area of a Figure*** is the *quantity* of *surface* it contains. It is often called the *superficial contents.*

NOTE.—The term *rectangular* signifies *right angled.*

1. How many square feet of canvas in a rectangular painting 3 feet long and 2 feet wide?

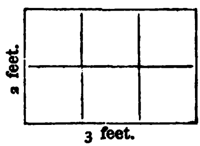


ILLUSTRATION.—Let the painting be represented by the figure in the margin; its length being divided into *three* equal parts, and its breadth into *two*; each denoting a *linear* foot. It will be seen that there are 2 rows of squares in the figure, and 3 squares in a row. Therefore, the painting must contain 2 times 3, or 6 square feet.

8. How find the *area* of a rectangular surface?

Multiply the length and breadth together.

2. How many square feet in a blackboard 8 ft. long and $3\frac{1}{2}$ ft. wide?

3. How many square inches in a pane of glass 32 in. long and 24 in. wide?

4. How many square rods in a garden 15 rods long and 6 rods wide?

5. How many yards of carpeting 1 yard wide are required to cover a room 18 feet long and 15 feet wide?

6. How many sq. feet in a board 16 ft. long and $1\frac{1}{2}$ ft. wide?

7. How many acres in a farm 100 rods long and 80 rods wide?

8. How many acres in a township 6 miles square?

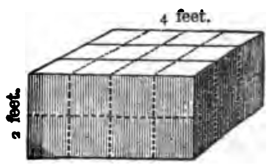
9. A flower garden is 30 yards long and 18 yards wide: what are its contents?

10. How many brick 8 in. long and 4 in. wide, will it take to pave a sidewalk 40 ft. long and 5 ft. wide?

11. What is the cost of a pine board 18 ft. long and $2\frac{1}{2}$ ft. wide, at 8 cts. a square foot?

MEASUREMENT OF RECTANGULAR SOLIDS.

9. What is a Rectangular Body?



A ***Rectangular Body*** is one bounded by *six rectangular* sides, each *opposite* pair being *equal* and *parallel;* as, boxes of goods, blocks of hewn stone, etc.

When *all* the sides are equal, it is called a *cube.*

10. What are the Contents of a body?

The ***Contents*** or ***Solidity*** of a body is the *quantity* of *matter* or *space* it contains.

1. How many cu. feet are there in a box of books 4 ft. long, 3 ft. wide, and 2 ft. deep?

ILLUSTRATION.—Let the box be represented by the preceding figure; its length being divided into *four* equal parts, its breadth into *three,* and its depth into *two*; each part denoting a linear foot. In the upper surface of the box there are 3 times 4, or 12

sq. feet. Now, if the box were 1 foot deep, it would contain 1 time as many *cubic* feet as there are *square* feet in its upper face, and 1 time $4 \times 3 = 12$ cu. ft. But the box is 2 feet deep; therefore it must contain 2 times $4 \times 3 = 24$ cu. feet.

11. How find the contents of a *rectangular* body.

Multiply the length, breadth, and thickness together.

2. How many cu. inches in a brick 8 in. long, 4 in. wide, and 2 in. thick?

3. How many cubic feet in a box of sugar 5 ft. long, 3 ft. wide, and 3 ft. deep?

4. Henry made a pile of cubic letter blocks, the length of which was 8 blocks, the width 6 blocks, and the height 5 blocks: how many blocks were in the pile?

5. How many cubic feet in a pile of brick 13 ft. long, 7 ft. wide, and 5 ft. high?

6. How many cubic feet in a load of wood 7 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high?

7. How many cu. feet in a bin 12 ft. long, 6 ft. wide, and $5\frac{1}{4}$ ft. deep?

8. How many cu. yards of earth must be removed to dig a cellar 36 ft. long, 20 ft. wide, and 6 ft. deep?

9. How many cu. feet in a stick of timber 36 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{1}{2}$ ft. thick?

10. What will it cost to build a wall 120 ft. long, $1\frac{1}{2}$ ft. thick, and 9 ft. high, at 15 cts. a cubic foot?

11. What will it cost to dig a trench 100 ft. long, 9 ft. wide, and $4\frac{1}{2}$ ft. deep, at 27 cts. a cu. yard?

12. What is the worth of a pile of wood 48 ft. long, 6 ft. high, and 4 ft. wide, at $\$4\frac{1}{2}$ a cord?

13. A rectangular bin is 10 ft. long, 6 ft. wide, and 4 feet deep: what are its contents?

14. A load of wood is $7\frac{1}{2}$ feet long, 4 ft. wide, and 3 ft. high: what are its contents?

REDUCTION OF DENOMINATE FRACTIONS.

12. What is a Denominate Fraction?

A ***Denominate Fraction*** is *one or more* of the equal parts into which a *Compound* or *Denominate* number may be divided.

13. How are they expressed?

Denominate Fractions are expressed either as *common* fractions, or as *decimals;* as, $\frac{3}{4}$ pound, .8 yard.

To reduce Denominate Fractions to *Units* of Lower Denominations.

1. Reduce $\frac{2}{5}$ gallon to quarts and pints.

ANALYSIS.—Since there are 4 qts. in a gal., there must be 4 times as many quarts as gallons; and 4 times $\frac{2}{5}$ gal. are $\frac{8}{5}$, equal to 1 qt. and $\frac{3}{5}$ qt. rem. Again, since there are 2 pt. in a quart, there must be 2 times $\frac{3}{5}$ or $\frac{6}{5}$ pt., equal to $1\frac{1}{5}$ pt. Therefore, etc.

OPERATION.

$\frac{2}{5}$ g. $\times 4 = \frac{8}{5}$, or 1 qt. and $\frac{3}{5}$ qt. rem.

$\frac{3}{5}$ qt. $\times 2 = \frac{6}{5}$, or $1\frac{1}{5}$ pt.

Ans. 1 qt. $1\frac{1}{5}$ pt.

14. How reduce denominate fractions to units of a lower denomination?

I. *Multiply the given numerator by the number required to reduce the fraction to the next lower denomination, and divide the product by the denominator.* (P. 173, Q. 4.)

II. *Multiply and divide the successive remainders in the same manner till the lowest denomination is reached. The several quotients will be the answer required.*

2. Reduce $\frac{5}{8}$ of a yard to feet and inches.
3. Reduce $\frac{3}{8}$ of a pound sterling to shillings and pence.
4. Reduce $\frac{9}{12}$ of a week to days and hours.
5. What part of a pint is $\frac{3}{48}$ of a gallon?

SOLUTION.—This example is the same in principle as the preceding. Reducing the numerator to the required denomination, place it over the givon denominator: $\frac{3}{48}$ gal. $\times 4 \times 2 = \frac{24}{48}$, or $\frac{1}{2}$ pt.

6. What part of a quart is $\frac{2}{96}$ of a bushel?

7. What part of a pennyweight is $\frac{1}{360}$ pound Troy?

8. Reduce .6 yard to feet and inches.

ANALYSIS.—Reducing .6 yard to feet, we have .6 yd. × 3 = 1 ft. and .8 ft. over. Again, reducing .8 ft. to inches, we have .8 ft. × 12 = 9.6 in. Therefore, .6 yard equals 1 ft. 9.6 in., which is the answer required. *Ans.* 1 ft. 9.6 in.

$$\begin{array}{r} .6 \text{ yd.} \\ \underline{3} \\ 1.8 \text{ ft.} \\ \underline{12} \\ 9.6 \text{ in.} \end{array}$$

15. How reduce a denominate decimal to units of lower denominations?

I. *Multiply the denominate decimal by the number required of the next lower denomination to make one of the given denomination, and point off the product as in multiplication of decimals.*

II. *Proceed in this manner with the decimal part of the successive products, as far as required. The integral part of the several products will be the answer.*

NOTE.—The preceding operations in Denominate Fractions are the same in principle as those in *Reduction Descending*.

9. Reduce .84 gal. to quarts and pints.

10. Reduce .625 week to days, etc.

11. Reduce .875 bushel to pecks, etc.

To reduce a Compound Number from a Lower to a Denominate Fraction of a Higher Denomination.

12. What part of a gallon is 1 pint and 2 gills?

ANALYSIS.—1 pt. 2 gi. = 6 gills; and 1 gallon = 1 × 4 × 2 × 4 = 32 gills. But 6 gills are $\frac{6}{32}$ of 32 gi., equal to $\frac{3}{16}$ gal. Therefore, etc.

1 pt. 2 gi. = 6 gi.
1 gal. × 4 × 2 × 4 = 32 gi.
Ans. $\frac{6}{32}$, or $\frac{3}{16}$ gal.

16. How reduce a compound number to a denominate fraction of a higher denomination?

I. *Reduce the given number to its lowest denomination for the numerator.*

II. *Reduce to the same denomination, a unit of the required fraction, for the denominator.*

NOTE.—If the lowest denomination of the given number contains a fraction, the number must be reduced to the parts indicated by the denominator of the fraction. (Ex. 16.)

13. Reduce 2 ft. 5 in. to the fraction of a yard.

14. Reduce 3 qt. 1 pt. to the fraction of a bushel.

15. What part of a pound sterling is 12s. 6d.?

16. What part of a pound Troy is $\frac{3}{5}$ pennyweight?

SOLUTION—The lowest denomination is 5ths of a pwt. Now 1 lb. Troy$=1\times12\times20\times5=1200$ fifths pwt. *Ans.* $\frac{3}{1200}$, or $\frac{1}{400}$ lb.

17. What part of a mile is $\frac{3}{4}$ of a rod?

18. Reduce $\frac{3}{5}$ of a quart to the fraction of a bushel?

19. What decimal part of a gallon is 3 qt. 1 pt. 2 gi.?

OPERATION.

$$\begin{array}{r|l} 4 & 2\text{ gi.} \\ \hline 2 & 1.5\text{ pt.} \\ \hline 4 & 3.75\text{ qt.} \\ \hline \end{array}$$

Ans. 0.9375 gal.

ANALYSIS.—Since 4 gi. are 1 pt., there must be 1 fourth as many pints as gills, and $\frac{1}{4}$ of 2 gi.$=\frac{2}{4}$, or .5 pt. Place the .5 pt. on the right of the given pints. Again, 2 pt. are 1 qt.; hence, there is 1 half as many quarts as pints; and $\frac{1}{2}$ of 1.5 pt.$=$.75 qt., which we place on the right of the given quarts. Finally, 4 qt. are 1 gal.; hence, there is 1 fourth as many gallons as quarts; and $\frac{1}{4}$ of 3.75 qt. $=$0.9375 gal. Therefore, etc.

17. How reduce a compound number to a denominate decimal of a higher denomination.

I. *Write the numbers in a column, placing the lowest denomination at the top.*

II. *Beginning with the lowest, divide it by the number required of this denomination to make a unit of the next higher, and annex the quotient to the next higher.*

Proceed in this manner with the successive denominations, till the one required is reached.

20. Reduce 3 fur. 20 rods to the decimal of a mile.

21. What decimal of a pound is 6s. 8d.?

22. Reduce 5 gal. 3 qt. to the decimal of a hogshead.

COMPOUND ADDITION.

1. What is the sum of 5 gal. 3 qt. 1 pt. 3 gi.; 8 gal. 2 qt. 1 pt. 1 gi.; 4 gal. 3 qt. 1 pt. 3 gi.?

OPERATION.

	Gal.	qt.	pt.	gi.
	5	3	1	3
	8	2	1	1
	4	3	1	3
Ans.	19	2	0	3

ANALYSIS.—Write the numbers so that the same denominations shall stand in the same column, and beginning at the right, add the columns separately. Thus, 3 gi. and 1 gi. are 4 gills, and 3 are 7 gills, equal to 1 pt. and 3 gi. Set the 3 gi. under the column of gills, and carrying the 1 pt. to the column of pints, the sum is 4 pints, equal to 2 qts. and no remainder. Write a cipher under the pints, and carrying the 2 qts. to the column of quarts, the sum is 10 qts., equal to 2 gal. and 2 quarts. Set the 2 qts. under the quarts, and carrying the 2 gal. to the column of gallons, the sum is 19 gals. Hence, 19 gal. 2 qt. 0 pt. 3 gi. is the sum required.

1. How add Compound Numbers?

I. *Write the same denominations one under another, and beginning at the right, add each column separately.*

II. *If the sum of a column is less than a unit of the next higher denomination, write it under the column added.*

If equal to one or more units of the next higher denomination, carry these units to that denomination, and write the excess under the column as in Simple Addition. (P. 28, Q. 13.)

REMARK.—Addition, Subtraction, etc., of Compound Numbers are the same in principle as the corresponding operations in Simple Numbers. The only difference between them arises from their *scales of increase*. The orders of the latter increase by the *constant scale* of 10. The denominations of the former increase by a *variable scale*. In both we carry for the number which it

takes of a lower order or denomination to make one in the next higher. In the former, this number is always 10; in the latter, it is *variable*.

(2.)

£.	s.	d.	far.
10	13	4	2
7	5	3	3
8	3	5	2

(3.)

Lb.	oz.	pwt.
13	8	9
8	6	8
5	8	3

(4.)

Yd.	ft.	in.
8	2	5
7	1	8
5	2	7

(5.)

Bu.	pk.	qt.	pt.
15	3	7	1
30	2	3	1
8	3	5	0
53	1	6	1

(6.)

T.	cwt.	lb.
5	18	35
8	3	83
3	17	35
7	5	70

(7.)

M.	fur.	r.
35	3	28
84	5	15
38	3	8
27	4	13

8. In one bin there are 35 bu. 3 pk. and 7 qts. of oats; in another, 27 bu. 2 pk. 5 qts.; and in another, 28 bu. 3 pk: how many bushels are there in all?

9. A merchant sold 3 pieces of muslin: one containing 35 yds. and 1 fourth; another, 43 yds. and 5 eighths; and the other, 38 yds. and 3 eighths: how many yards did he sell?

10. In one garden there are 13 sq. r. 5 sq. yd. 3 sq. ft.; in another, 18 sq. r. 8 sq. yd. 5 sq. ft.; and in another, 23 sq. r. 5 sq. yd. and 8 sq. ft.: how much land in all?

11. A farmer has 4 fields: one containing 18 A. 35 sq. rods; another 30 A. 78 sq. r.; another 45 A. 30 sq. r., the other, 23 A. 65 sq. r.: how many acres has he?

12. How much wood is there in 3 loads, one of which contains 1 C. 45 cu. ft.; another, 1 C. and 58 cu. ft.; and the other 1 C. 85 cu. ft.?

COMPOUND SUBTRACTION.

1. From 27 yd. 1 ft. 8 in., take 18 yd. 2 ft. 5 in.

OPERATION.			
	Yd.	ft.	in.
	27	1	8
	18	2	5
Ans.	8	2	3

ANALYSIS.—Write the less number under the greater, placing the same denominations in the same column. Beginning at the right, we proceed thus: 5 in. from 8 in. leave 3 in.; set the 3 under the column of inches. Next, since 2 ft. cannot be taken from 1 ft., we borrow a unit of the next higher denomination, which is yards. Now 1 yd. or 3 ft. added to 1 ft. make 4 ft., and 2 ft. from 4 ft. leave 2 ft. Finally, 1 to carry to 18 makes 19, and 19 yds. from 27 yds. leave 8 yds. Hence, the difference is 8 yds. 2 ft. 3 in.

2. How subtract Compound Numbers?

I. *Write the several denominations of the subtrahend under those of the same name in the minuend.*

II. *Beginning at the right, subtract each denomination of the subtrahend from that above it, and set the remainder under the term subtracted.*

III. *If the number in any denomination of the subtrahend is larger than that above it, add to the upper number as many as are required to make a unit of the next higher; then subtract and carry 1 to the next denomination in the subtrahend, as in Simple Subtraction.*

	(2.)	(3.)
From	£25, 7s. 6d. 2 far.	13 lb. 7 oz. 18 pwt. 23 gr.
Take	$23, 5s. 3d. 3 far.	7 lb. 8 oz. 13 pwt. 18 gr.

4. From 2 bu. take 3 pk. 5 qt.

5. From 8 m. 130 r. take 250 r. 3 yd. 2 ft.

6. From a hogshead of molasses 35 gal. 3 qt. were drawn: how many gallons were left?

7. If one farm contains 165 A. 118 sq. r., and another 100 A. 135 sq. r., what is the difference between them?

8. What is the difference between two loads of wood, one of which contains 1 C. 38 cu. ft., the other 125 cu. ft.?

9. What is the difference in the weight of two stacks of hay, one of which contains 5 T. 135 lb., the other 7 T. 387 lb.?

10. The longitude of New York is 74° 0′ 3″ W.; that of Chicago, 87° 35′ W.: what is the difference in their longitude?

11. The latitude of New Orleans is 29° 57′ 30″ N.; that of Montreal 45° 31′ N.: what is the difference in their latitude?

12. What is the difference of time between Dec. 25th, 1865, and April 20th, 1872?

SOLUTION.—Place the earlier date under the later, the *years* on the left, the *months* next, and the *days* on the right, and proceed as in subtracting other Compound Numbers.

	Y.	m.	d.
	1872	4	20
	1865	12	25
Ans.	6	3	25

REMARK.—In finding the difference between two dates, and in most business transactions, 30 days are considered a *month*, and 12 months a *year*.

13. If a man was born Jan. 1st, 1850, how old will he be July 4th, 1876?

14. A note dated March 13th, 1870, was paid Feb. 25th, 1872: how long did it run?

15. Charles was born July 30th, 1865, and his brother Oct. 24th, 1869: what is the difference in their ages?

16. A whale-ship started on a voyage Aug. 25th, 1867, and returned July 18th, 1871, how long was she gone?

COMPOUND MULTIPLICATION.

1. If a family uses 2 lbs. 12 oz. of butter a day, how much will they use in 3 days?

ANALYSIS.—They will use 3 times 2 lb. and 12 oz. Now 3 times 2 lb. are 6 lb., and 3 times 12 oz. are 36 oz., equal to 2 lb. 4 oz.; which, added to 6 lb., make 8 lb. 4 oz. Therefore, etc.

2. If it takes 4 gal. 3 qt. of water to fill a demijohn, how much will it take to fill 2 of the same size?

3. A farmer gave a bag of corn, containing 2 bu. 3 pk., to each of 4 beggars: how much did he give to all?

4. If it takes 3 yd. 1 qr. of cloth to make a boy's suit, how many yards will it take to make 5 suits?

5. How long will it take a man to chop 3 cords of wood, if he chops at the rate of a cord in 4 hr. 30 min.?

6. If you pick 2 qt. 1 pt. of blackberries an hour, how many can you pick in 6 hours?

7. If 1 book costs 2 shillings and 6 pence, what will 5 books cost?

SLATE EXERCISES.

1. A miller ground 5 grists, each containing 2 bushels, 3 pecks, 5 quarts of wheat: how much wheat did he grind?

OPERATION.

Bu.	pk.	qt.
2	3	5
		5
14	2	1

ANALYSIS.—5 grists contain 5 times as much as 1 grist. Now 5 times 5 qt. are 25 qt., equal to 3 pk. and 1 qt. Set the 1 qt. under the quarts, and carry the 3 pk. to the product of pecks. Next, 5 times 3 pk. are 15 pk., and 3 are 18 pk., equal to 4 bu. and 2 pk. Set the 2 under the pecks, and carrying the 4 bu. to the product of bu. we have 5 times 2 are 10 bu., and 4 are 14 bu. Therefore, he ground 14 bu. 2 pk. 1 qt.

3. How multiply Compound Numbers?

I. *Write the multiplier under the lowest denomination of the multiplicand, and beginning at the right, multiply each term in succession.*

II. *If the product of any term is less than a unit of the next higher denomination, set it under the term multiplied.*

III. *If equal to one or more units of the next higher denomination, carry these units to that denomination, and write the excess under the term multiplied.*

(2.)

	R.	yd.	ft.
Mult.	13	2	2
By			5
Ans.	67	2	1

(3.)

M.	fur.	r.	yd.
30	3	18	4
			8
243	3	29	$4\frac{1}{2}$

4. Bought 5 casks of vinegar, each containing 36 gal. 3 qt. 1 pt.: how much did they all contain?

5. Sold 6 pieces of cloth, each containing 42 yards and 3 quarters: how much did all contain?

6. A farmer has 4 pastures, of 15 A. 63 sq. r. each: how much land in all?

7. A man bought 10 loads of wood, each containing 1 C. 35 cu. ft.: how much wood did he buy?

8. If you read 5 h. 35 min. per day, how many hours will you read in 12 days?

9. Bought 7 loads of hay, averaging 1 T. 375 lbs.: how much did all contain?

10. If the price of one cow is £8, 15s. $6\frac{1}{4}$d., what will 8 cows cost, at the same rate?

11. If you have 11 apple-trees, and they yield 7 bu. 3 pk. apiece, how many apples will you have?

COMPOUND DIVISION.

1. If 48 lb. 12 oz. of rice are divided equally among 15 persons, what part, and how much, will each receive?

ANALYSIS.—1 is $\frac{1}{15}$ of 15; therefore, each person will receive 1 fifteenth part.

Again, 1 fifteenth of 48 lb. is 3 lb., and 3 remainder. Reducing the remainder 3 lb. to ounces, they become 48 oz., and adding the 12 oz. we have 60 oz. Now 1 fifteenth of 60 oz. is 4 oz. Therefore, each received $\frac{1}{15}$ part, which is 3 lb. 4 oz.

OPERATION.

	lb.	oz.
15)	48	12
Ans.	3	4

NOTE.—1. The object in this example is to divide a compound number into *equal parts*, in order to find the *value* of one part.

2. A farmer sent 29 bu. 1 pk. of wheat to mill, in bags of 3 bu. 1 pk. each: how many bags did he use?

ANALYSIS.—Reducing the whole quantity to pecks, it becomes 117 pk. The quantity in each bag, 3 bu. 1 pk., equal 13 pk. We now divide as in simple numbers.

29 bu. 1 pk. = 117 pk.
3 bu. 1 pk. = 13 pk.
13)117
Ans. 9 bags.

NOTE.—2. The object of this example is to find *how many times* one compound number is contained in another.

4. How divide Compound Numbers?

I. When the divisor is an *abstract* number,

Beginning at the left, divide each denomination in succession, and set the quotient under the term divided.

If there is a remainder, reduce it to the next lower denomination, and, adding it to the given units of that denomination, divide as before.

II. When the divisor is a *compound* number,

Reduce the divisor and dividend to the lowest denomination contained in either, and divide as in simple numbers.

REMARK.—It will be observed from the preceding examples, that the *object* of Compound as well as Simple Division is *twofold:*

1st, To divide a compound number into *equal parts*, the divisor being *abstract*. In this case the *quotient* is the same denomination as the *dividend.*

2d, To find how many times one compound number is contained in another. In this case the *quotient* is *times*, or an abstract number. (P. 63, Q. 10.)

Perform the following divisions:

(3.)

5) 16 A. 75 sq. r. 35 sq. ft.

(4.)

6) 505 cu. ft. 154 cu. in.

(5.)

2 lb. 4 oz.) 17 lb. 6 oz.

(6.)

£2, 12s.) £23, 8s.

7. If I sell 46 bu. 1 pk. of plums in equal quantities to 7 market-men, how many will each receive?

8. Charles having a kite-line 72 ft. 4 in. long, cut it into 7 equal parts: what was the length of each part?

9. If a man travels 48 m. 3 fur. in 9 hours, how far will he go in 1 hour?

10. A goldsmith made 5 lb. 3 oz. of silver into 24 spoons: what was the weight of each?

11. How many iron rails 18 ft. long are required to lay both sides of a track 7 m. 160 r. in length?

12. A man gathered 57 bu. 3 pk. of oranges from 9 trees: what was the average yield?

13. If 8 men mow 17 A. 32 sq. r. in a day, how much can 1 man mow?

14. How many times does a car-wheel 16 ft. 6 in. in circumference turn around in going 2 miles?

15. How many bags, holding 2 bu. 3 pk. each, can be filled from a bin which contains 19 bu. 1 pk. of corn?

16. How many bundles of hay, each weighing 465 pounds, can be made from a scaffold which contains 5 tons, 125 pounds?

PERCENTAGE.

1. What is meant by per cent?

Per cent denotes *hundredths.*

NOTE.—The term is from the Latin *per* and *centum, by* or *in a hundred.*

2. What is the rate?

The ***Rate*** is the number which shows how many *hundredths* are taken. Thus 1 per cent of a number is 1 hundredth part of that number; 2 per cent, 2 hundredths; 3 per cent, 3 hundredths, &c.

3. With what do the terms rate per cent, correspond?

The terms ***Rate per cent*** correspond with the terms of a fraction, the *denominator* of which is always 100, and the *numerator* the given rate.

4. How then may per cent be expressed?

By a ***Common or a Decimal*** Fraction.

5. How is per cent expressed by decimals?

Write the figures denoting the per cent in the FIRST TWO *places on the right of the decimal point, and the parts of 1 per cent in the succeeding places toward the right.*

1 per cent is written		.01	$\frac{1}{2}$ per cent is written		.005
6 per cent	"	.06	$\frac{1}{4}$ per cent	"	.0025
10 per cent	"	.10	$2\frac{1}{2}$ per cent	"	.025
100 per cent	"	1.00	$6\frac{1}{4}$ per cent	"	.0625
106 per cent	"	1.06	$33\frac{1}{3}$ per cent	"	.33$\frac{1}{3}$
125 per cent	"	1.25	$107\frac{1}{2}$ per cent	"	1.075

6. How many figures are required to express per cent?

Per cent denotes *hundredths;* therefore every per cent requires at least *two figures,* to express it decimally.

7. If the given per cent is less than 10, what must be done?

A ***cipher*** must be prefixed to the figure denoting it. Thus, 1 per cent is written .01; 3 per cent .03, etc.

NOTE.—When a given part of 1 per cent cannot be exactly expressed by *one* or *two decimal* figures, it is written as a *common* fraction, and annexed to the figures expressing *hundredths*, or the *per cent*. Thus, $4\frac{1}{3}\%$ is written $.04\frac{1}{3}$, instead of .043333+.

8. To what is 100 per cent of a number equal?

A ***hundred per cent*** of a number is equal to the number itself; for $\frac{100}{100}$ is equal to 1.

9. When the rate is 100 per cent or over, how is it expressed?

By a ***mixed number,*** or by an *improper fraction*. Thus 125% is written 1.25, or $\frac{125}{100}$.

10. What is the sign of per cent?

The ***Sign of per cent*** is an oblique line between two ciphers (%). Thus, 2% is read 2 per cent, etc.

Write the following per cents decimally:

1. 4%; 7%; 10%; 45%; 103%; 110%; 205%.
2. $2\frac{1}{2}\%$; $6\frac{1}{2}\%$; $7\frac{1}{2}\%$; $18\frac{3}{4}\%$; $106\frac{1}{4}\%$; $112\frac{1}{2}\%$.

11. How read a given per cent expressed decimally?

Read the first two decimal figures as per cent, and those on the right as decimal parts of 1 per cent.

Copy and read the following as rates per cent:

1. .03; .06; .045; $.11\frac{1}{3}$; .625; 1.25; 1.50; 2.00.
2. 1.06; 1.07; 1.08; $1.10\frac{1}{2}$; $1.62\frac{1}{2}$; 1.005; 2.00.

12. What are the elements or parts in calculating percentage?

The *base*, the *rate per cent*, the *percentage*, and the *amount*.

13. Explain each.

The ***Base*** is the *number* on which the *percentage* is calculated.

The ***Rate per cent*** is the *number* which shows *how many hundredths* of the *base* are to be taken.

The ***Percentage*** is the *number obtained* by taking that portion of the *base* indicated by the rate *per cent.*

The ***Amount*** is the *base*, increased or diminished by the *percentage.*

NOTE.—The conditions of the question show whether the *percentage* is to be *added* to, or *subtracted* from the *base* to form the *amount*.

CASE I.

To find the *Percentage*, the Base and Rate being given.

MENTAL EXERCISES.

1. How many are $\frac{3}{5}$ of 40? (P. 90.)

ANALYSIS.—1 fifth of 40 is 8, and 3 fifths are 3 times 8, or 24.

2. How many are 40 multiplied by $\frac{3}{5}$? *Ans.* 24.

3. To how many hundredths is $\frac{3}{5}$ equal?

ANALYSIS.—$1 = \frac{100}{100}$; hence $\frac{1}{5}$ equals $\frac{1}{5}$ of $\frac{100}{100}$, or $\frac{20}{100}$; and 3 fifths = 3 times $\frac{20}{100}$, or $\frac{60}{100}$. (P. 95, Q. 20.)

4. What is $\frac{60}{100}$ of 40? *Ans.* 24. (P. 90.)

5. To what per cent is $\frac{60}{100}$ equal? *Ans.* 60 per cent.

6. What is 60 per cent of 40?

ANALYSIS.—60% is 60 times 1%. Now, 1% of 40 is $\frac{1}{100}$ of 40, or $\frac{40}{100}$, and 60 times $\frac{40}{100} = \frac{2400}{100}$, or 24, *Ans.*

7. To what per cent is $\frac{5}{100}$ equal? $\frac{7}{100}$? $\frac{10}{100}$? $\frac{15}{100}$?

8. Reduce $\frac{1}{4}$ to hundredths. $\frac{3}{5}$ to hundredths. $\frac{7}{20}$ to hundredths.

9. To what per cent of a number is $\frac{1}{5}$ of it equal?

ANALYSIS.—$\frac{1}{5}$ equals $\frac{20}{100}$; therefore, $\frac{1}{5}$ of a number is 20%. (P. 95, Q. 20.)

10. To what per cent is $\frac{1}{4}$ equal? $\frac{3}{4}$? $\frac{7}{10}$? $\frac{3}{25}$?

11. What is 5 per cent of 200 yards?

ANALYSIS.—5% is the same as $\frac{5}{100}$. Now, $\frac{1}{100}$ of 200 yards is 2 yards, and 5 hundredths are 5 times 2, or 10 yards. Therefore, 5% of 200 yards is 10 yards.

12. What is 7% of $300? 8% of 500 barrels?

13. Which is the greatest, $\frac{3}{5}$ of 200; or 200 × $\frac{3}{5}$; or 200 × .60; or 60 per cent of 200?

SLATE EXERCISES.

REMARK.—From the preceding illustrations, it will be seen that *finding* a *fractional part* of a number, *multiplying* it by a common or a *decimal fraction*, and *finding a per cent* of it, are *identical in principle.* With the *first two* the learner is supposed to be familiar; if not, he should *carefully review* them before going further. (P. 90, 113, 139.)

1. What is 4% of $315?

ANALYSIS.—4 per cent is the same as $\frac{4}{100}$, and $\frac{4}{100}$ expressed decimally is .04; therefore, 4 per cent of $315 is .04 times $315. Multiplying the base by the rate, expressed decimally, we have $315 × .04 = $12.60, the percentage required.

OPERATION.

$$\begin{array}{r} \$315\ \text{B.} \\ .04\ \text{R.} \\ \hline \textit{Ans.}\ \$12.60\ \text{P.} \end{array}$$

14. How find the *Percentage*, when the base and rate are given?

RULE.—*Multiply the base by the rate, expressed decimally.*

NOTES.—1. When the *rate* is an *even* part of 100, the *percentage* may be found by taking a *like part* of the base. Thus, for 20%, take $\frac{1}{5}$; for 25%, take $\frac{1}{4}$, etc. (Ex. 3.)

2. The *amount* is found by *adding the percentage to,* or *subtracting it from the base,* as the case may be. (Ex. 10.)

2. What is 6% of $415.50? *Ans.* $24.93.

3. What is 25% of 460 pounds?

SOLUTION.—460 pounds × $\frac{1}{4}$ = 115 pounds, *Ans.*

4. 5% of 640 yards.
5. 6% of $765.60.
6. 7% of 600 bushels.
7. 8% of 1000 rods.
8. 12% of 1110 barrels.
9. 20% of 2040 men.

10. A farmer having 163 acres of land, bought 12% more, how many acres did he then own?

SOLUTION.—163 A. × .12 = 19.56 A. bought; 163 A. + 19.56 A. = 182.56 A. owned.

11. A man having 560 sheep, lost $2\frac{1}{2}\%$ of them by sickness: how many did he have left?

ANS.—560 s. × $.02\frac{1}{2}$ = 14 s. lost; 560 s. − 14 s. = 546 s. left.

12. A teacher's salary is 12% more this year than last; it was then $1500: what is it now?

13. From a school of 750 pupils, 20 per cent were absent: how many were present?

14. What is $62\frac{1}{2}\%$ of $25000?

CASE II.

To find the *Rate*, the Base and Percentage being given.

1. What part of 4 is 3?

ANALYSIS.—1 is $\frac{1}{4}$ of 4, and 3 must be 3 times 1 fourth, or 3 fourths of 4.

2. What part of 5 is 3? What part is 4?

3. To how many hundredths is $\frac{3}{4}$ equal?

4. What part of 100 is 4? What part is 5? 7? 9?

5. What per cent of a number is $\frac{5}{100}$? $\frac{9}{100}$? $\frac{25}{100}$?

6. What per cent of $5 are $2?

ANALYSIS.—$2 are $\frac{2}{5}$ of $5; and $\frac{2}{5}$ equals $\frac{40}{100}$; therefore, $2 are 40% of $5. (P. 95, Q. 20.)

7. What per cent of 10 yards are 3 yards? 5 yds.? 7 yds.?

8. What part of a number is 20%, expressed in the lowest terms of a common fraction?

ANALYSIS.—20 per cent = $\frac{20}{100}$; and $\frac{20}{100} = \frac{1}{5}$ *Ans.* (P. 95.)

9. What part of a number is 25%, expressed by decimals?

10. What part of a number is 5 per cent? 10 per cent? 40 per cent?

SLATE EXERCISES.

Remark.—From the preceding illustrations, it will be seen that *finding the rate*, when the base and percentage are given, is the *same in principle* as *finding what part* one number is of another; then *changing the common fraction to hundredths.*

1. A man paid $28 for a cow, and sold it so as to make $7: what per cent did he make?

Operation.

$$28\ )\ 7.00$$

Ans. 25%

Analysis.—In this example the base $28, and the percentage $7, are given, to find the rate per cent. Now, $7 are $\frac{7}{28}$ of $28; and $\frac{7}{28} = 7 \div 28 = .25$ or 25%.

Or $\frac{7}{28} = \frac{1}{4} = \frac{25}{100}$, or 25%. (P. 95, Q. 20.)

15. How find the *Rate*, when the Base and Percentage are given?

Divide the percentage by the base; the first two decimal figures will be hundredths, or the rate per cent; the others, parts of one per cent.

Note.—The number denoting the *base* is always preceded by the word *of*, which distinguishes it from the *percentage*.

2. A lady having $40, spent $7 for a collar: what per cent of her money did she spend? *Ans.* $17\frac{1}{2}$%.

3. What % of 18 is 6?
4. What % of £1 are £3?
5. What % of $35 are $7?
6. What % of 54 is 6?
7. What % of 150 is 75?
8. What % of 135 is 4.5?
9. What % of 150 is 30?
10. What % of $\frac{3}{4}$ is $\frac{1}{8}$?
11. What % of .8 is .2?
12. What % of $\frac{4}{5}$ is $\frac{2}{10}$?

13. A man bought a farm of 200 acres, and sold 50 acres of it: what per cent of his farm did he sell?

14. A farmer having 500 sheep, sold 125 of them: what per cent of his flock did he sell?

15. If a man earns $450 a year, and lays up $225 of it, what per cent of his earnings does he save?

16. From a school of 750 pupils, 250 were absent: what per cent were absent?

APPLICATIONS OF PERCENTAGE.

1. To what classes of problems is percentage applied?

First, To those in which *time* is one of the elements of calculation; as Interest, etc.

Second, To those which are *independent* of time; as Commission, Profit and Loss, etc.

COMMISSION.

2. What is Commission, and how computed?

Commission is an allowance made to Agents, Collectors, etc., for the transaction of business, and is computed like *percentage.*

NOTES.—1. An *Agent* is one who transacts business for another, and is often called a *Commission Merchant.*

2. A *Collector* is one who collects debts, taxes, duties, etc.

3. Goods sent to an agent to sell, are called a *consignment;* the person to whom they are sent, the *Consignee;* and the person sending them, the *Consignor.*

3. What answers to the base, the rate, etc.?

The *amount of sales,* etc., is the *base.*

The *per cent for services,* the *rate.*

The *commission,* the *percentage.*

The *amount of sales,* etc., *plus* or *minus* the *commission,* the *amount.*

To find the *Commission,* the Amount of Sales and the Rate being given.

1. A merchant sold goods to the amount of \$250, at 3% commission: what was his commission?

ANALYSIS.—In this example the amount of sales \$250, is the base, and 3% the rate. The question then is, what is 3% of \$250. Now, 3% equals .03 expressed decimally; therefore, 3% of \$250 is .03 times \$250, and \$250 × .03 = \$7.50, the commission required.

OPERATION.

\$250 B.
.03 R.
Ans. \$7.50 C.

2. A broker sold 3 shares of bank stock for \$300: what was his commission at $\frac{1}{2}$ per cent?

SOLUTION.—$\frac{1}{2}\% = .005$, and $\$300 \times .005 = \$1.50$, *Ans.*

4. How find the *Commission*, when the amount of sales and rate are given?

Multiply the amount of sales by the rate, expressed decimally. (P. 196, Q. 14.)

REMARKS.—1. The *net proceeds* of a business transaction, are the *gross amount* of sales, etc., *minus* the commission and other charges.

2. When the *amount* of sales, etc., and the *commission* are known, the *net proceeds* are found by *subtracting the commission* from the *amount* of sales. (Ex. 5.) Conversely,

3. When the *net proceeds* and *commission* are known, the *amount* of sales is found by *adding the commission* to the *net proceeds*. (Ex. 6.)

3. A man collected a school tax of \$1250, at 5% commission: how much did he receive?

4. If you sell a consignment of goods for \$1175.60, what will be your commission at 4%?

5. My agent sold a quantity of flour for \$1585, and charged me 4%: what was his commission, and what the net proceeds?

ANALYSIS.—$\$1585 \times .04 = \$63.40$ com.; $\$1585 - \$63.40 = \$1521.60$, net proceeds.

6. Received \$115.20 for selling a consignment of goods, and sent the consignor \$2444.80: what was the amount of sales, and what % was my commission?

ANALYSIS.—$\$2444.80 + \$115.20 = \$2560$ sales; $115.20 \div 2560 = .04\frac{1}{2}\%$ commission.

7. My agent bought 28 shares of N. Y. Central R. R. for \$2800, and charged me $1\frac{1}{4}$ per cent commission: what was his commission?

PROFIT AND LOSS.

5. What are Profit and Loss, and how computed?

Profit and Loss are sums *gained* or *lost* in business transactions, and are computed like *Percentage.*

6. What answers to the base, the rate, etc.?

The *Cost* or *sum invested* is the *Base;*
The *Per cent* profit or loss, the *Rate;*
The *Profit or Loss,* the *Percentage;*
The *Selling Price,* that is, the cost plus or minus the profit or loss, the *Amount.*

To find the *Profit or Loss,* the Cost and the Rate per cent Profit or Loss being given.

1. Bought a horse for $150, and sold it for 12 per cent profit: how much did I gain by the transaction?

ANALYSIS.—In this case the cost $150 is the base, and 12% the rate. Now, 12% is the same as .12, and .12 times $150 = $150 × .12 = $18.00. Therefore I gained $18.

$$\begin{array}{r} \$150\ \text{B.} \\ .12\ \text{R.} \\ \hline \textit{Ans.}\ \$18.00\ \text{G.} \end{array}$$

2. Bought a carriage for $250, and sold it at a loss of 8%: how much did I lose?

ANALYSIS.—Here the cost $250 is the base, and 8% the rate. Now, 8% is the same as .08, and .08 times $250 = $250 × .08 = $20.00. Therefore, my loss was $20.

$$\begin{array}{r} \$250\ \text{B.} \\ .08\ \text{R.} \\ \hline \textit{Ans.}\ \$20.00\ \text{L.} \end{array}$$

7. How find the *Profit or Loss,* when the cost and rate are given?

Multiply the cost by the rate expressed decimally, as in percentage. (P. 196, Q. 14.)

REMARKS.—1. When the *per cent* is an *even part* of 100 it is generally *shorter,* and therefore *preferable* to use the *fraction.*

2. The *selling price* is found by adding the *profit* to or subtracting the *loss* from the cost, as the case may be. (Ex. 9.)

3. A man bought a cow for \$35, and sold it at 20% profit: how much did he gain?

4. Henry bought a pair of skates for \$3.75, and sold them for 20% more than he gave: what was his gain?

5. A grocer bought flour at \$8.50 a barrel, and sold it at 25% loss: what did he lose on a barrel?

6. A merchant bought a piece of silk for \$585, and sold it for 18% advance: what was his gain?

7. Bought a house lot for \$230, and sold it at 8% less than cost: what was my loss?

8. A man paid \$260 for a buggy, and sold it at 25% profit: how much did he make by the operation?

9. If a man pays \$150 for a watch, for what must he sell it to gain 12%?

ANALYSIS.—To gain 12% he must sell it for the *cost, plus* 12%. Now, 12% of \$150 = \$150 × .12 = \$18.00; and \$150 + \$18 = \$168, *Ans.*

10. A grocer paid \$200 for a lot of peaches, and finding them damaged, sold them at a loss of 25%: for what did he sell them?

ANALYSIS.—To lose 25% he must sell them for the *cost, minus* 25%. Now, 25% of \$200 is \$50; and \$200 − \$50 = \$150, the selling price.

11. Bought a case of 12 hats at \$6.50 apiece: for what must I sell the whole to make 20%?

12. A dealer bought 50 buffalo robes, at \$8 apiece, and sold them at 16% loss: what did he get for them?

13. A merchant bought 240 barrels of flour, at \$7.50 a barrel, and sold it at a profit of $12\frac{1}{2}$%: what did the flour come to?

14. Bought a farm for \$4200, and sold it for $33\frac{1}{3}$% more than cost: for how much was it sold?

To find the *per cent* Profit or Loss, the cost and the amount of Profit or Loss being given.

1. I bought a sleigh for $50, and sold it for $20 more than the cost: what per cent was the profit?

ANALYSIS.—The gain $20 is $\frac{20}{50}$ of $50, the cost; now $\frac{20}{50} = \frac{40}{100}$ or 40 per cent. (P. 198, Q. 15.) Therefore the profit was 40 per cent.

8. How find the *rate per cent*, the cost and the amount of profit or loss being given?

Divide the amount of profit or loss by the cost, as in percentage. (P. 198, Q. 15.)

REMARK. — When the *cost* and *selling price* are given, the *Profit* is found by subtracting the *cost* from the *selling* price. The *Loss* is found by subtracting the *selling price* from the *cost.*

2. Bought a horse for $200, and sold it for $50 less than cost: what per cent was the loss?

3. A fruit dealer bought oranges at 4 cents, and sold them so as to make 2 cents on each: what per cent was his profit?

4. If you buy a slate for 10 cents, and sell it for 5 cents more than cost: what per cent is your gain?

5. Paid $35 for a cow, and sold her for $10 less than cost: what per cent was the loss?

6. If a man buys flour at $7.50 a barrel, and sells it for $8.50, what per cent is his profit?

7. If you buy tea at 60 cents a pound, and sell it at 80 cents, what per cent is your profit?

8. If a grocer buys sugar at 8 cents a pound, and sells it at 6 cents, what per cent is his loss?

9. A fruit dealer bought bananas at $3.50 a hundred, and sold them at $5 a hundred: what per cent did he make?

INTEREST.

1. What is interest?

Interest is a compensation for the use of money.

2. What are the elements or parts to be considered?

The *Principal*, the *Rate*, the *Interest*, the *Time*, and the *Amount*.

3. Explain each.

The ***Principal*** is the money lent.

The ***Rate*** is the per cent *per annum*.

The ***Interest*** is the percentage.

The ***Time*** is the period for which the principal draws interest.

The ***Amount*** is the sum of the principal and interest.

NOTE.—The term *per annum*, from the Latin *per* and *annus*, signifies *by the year*.

To find the interest of $1, at 6 per cent for months.

REMARK.—The learner should observe that Interest differs from the preceding applications of Percentage by introducing *time* as an *element* in connection with the *rate per cent*. The terms *rate* and *rate per cent* always mean a certain number of hundredths *yearly*, and *pro rata* for longer or shorter periods.

1. If I charge 6% yearly for the use of $1, how much shall I receive?

ANALYSIS.—6 per cent is $\frac{6}{100}$, and $1 is 100 cents. Now, $\frac{6}{100}$ of 100 cents is 6 cents. Therefore, I shall receive 6 cents.

2. If the interest of $1 for 1 year is 6 cents, how much is it for 1 month? For 2 months? For 3 months?

ANALYSIS.—Since the interest of $1 at 6 per cent for 12 months (1 year) is 6 cents, for 1 month it is 1 twelfth of 6 cents, and $\frac{1}{12}$ of 6 cents is $\frac{6}{12}$ or $\frac{1}{2}$ cent. Again, since the interest of $1 at 6% is $\frac{1}{2}$ cent for 1 month; for 2 months it is 2 halves or 1 cent; for 3 months, 3 halves or $1\frac{1}{2}$ cent; for 6 mos., 6 halves or 3 cents, etc.

4. What then is the interest of \$1 at 6 per cent, for any number of months?

The interest of \$1, at 6 per cent, for any number of months, is half as many cents as months.

4. What is the interest of \$1, at 6% for 5 months?

5. What is the interest of \$1, at 6% for 7 months?

To find the interest of \$1, at 6% for days.

6. What is the interest of \$1, at 6% for 1 d.? For 2 d.? 3 d.? 4 d.? 5 d.? 6 d.? etc.

ANALYSIS.—Since the interest of \$1 for 30 days (1 month), is $\frac{1}{2}$ cent, or 5 mills, for 1 day it is 1 thirtieth of 5 mills, and $\frac{1}{30}$ of 5 is $\frac{5}{30}$, or $\frac{1}{6}$ mill. Again, since the interest of \$1 for 1 day is $\frac{1}{6}$ mill, for 2 days, it is $\frac{2}{6}$ mill; for 3 days, $\frac{3}{6}$; for 6 days, $\frac{6}{6}$, or 1 mill.

5. What then is the interest of \$1, at 6 per cent for any number of days?

The interest of \$1, at 6 per cent, for any number of days, is 1 sixth as many mills as days.

7. What is the interest of \$1, at 6% for 19 days?

8. What is the interest of \$1, at 6% for 15 days? For 23 d.? 25 d.?

To find the interest of \$1, at 6 per cent for months and days.

9. What is the interest of \$1, at 6% for 4 m. 21 d.?

ANALYSIS.—The interest of \$1 for 4 m. is $\frac{1}{2}$ of 4 or 2 cents; and the interest of \$1 for 21 d. is $\frac{1}{6}$ of 21 or $3\frac{1}{2}$ mills, which, added to 2 cts., make 2 cts. + $3\frac{1}{2}$ mills, or \$.0235.

6. How find the interest of \$1, at 6% for months and days.

Take half the number of months for cents, and one sixth the number of days for mills. The sum will be the interest.

NOTE.—In finding 1-sixth of the days, it is commonly sufficient to carry the *quotient* to *tenths* or *hundredths* of a mill.

10. What is the interest of \$1, at 6% for 6 m. 24 d.?

11. What is the interest of \$1, at 6% for 8 m. 27 d.?

General Method of Computing Interest, the Principal, the Time, and the Rate being given.

12. What is the interest of \$115.20 for 1 y. 1 m. and 12 d., at 6% ?

ANALYSIS. — 1 year equals 12 m.,		OPERATION.
and 12 m. + 1 m. . .	= 13 m.	\$115.20
The int. of \$1 for 13 months .	= \$.065	.067
" " 12 days . .	= .002	\$80640
The int. of \$1 for 1 y. 1 m. and 12 d.	= \$.067	69120
		\$7.71840

Since the int. of \$1 for the given time is \$.067, or .067 times the principal, the int. of \$115.20 must be .067 times that sum, and \$115.20 × .067 = \$7.7184, *Ans.*

13. What is the interest of \$150 for 10 months, at 7 per cent?

ANALYSIS.—At 6% the int. of \$150 for the time is \$150 × .05 = \$7.50. But 7% is 1%, or $\frac{1}{6}$ more than 6% ; and $\frac{1}{6}$ of \$7.50 is \$1.25. Now, \$7.50 + \$1.25 = \$8.75, the interest at 7 per cent.

$$\begin{array}{r} \$150 \\ .05 \\ \hline 6\,)\,\$7.50 \\ 1.25 \\ \hline \textit{Ans. } \$8.75 \end{array}$$

7. How find the interest on a given principal, for any given time and rate?

I. When the rate is 6 per cent,

Multiply the principal by the interest of \$1, at 6 per cent for the time, expressed decimally.

II. When the rate is greater or less than 6 per cent,

Add to or subtract from the interest at 6 per cent such a part of itself as the given rate exceeds or falls short of 6 per cent.

8. How find the amount?

Add the interest to the principal.

NOTES.—1. In finding the *time,* first determine the number of *entire calendar* months; then the *number* of days left.

2. In computing interest, if the *mills* are 5 or more, it is customary to add 1 to the cents; if less than 5, they are disregarded.

Only *three decimal* figures are retained in the following answers:

14. Find the amount of $75.60 for 1 y. 3 m. 9 d., at 6%.

SOLUTION.—Int. of $1 for the given time and rate, is $.0765. Now, $75.60 × .0765 = $5.7834, the int. The prin. $75.60 + $5.7834 = $81.383, *Amt.*

15. Find the int. of $45.50 for 1 y. 7 m. 15 d., at 7%.
16. Find the amt. of $58.75 for 1 y. 10 m. 21 d., at 6%.
17. The int. of $85 for 8 m. 9 d., at 5%.
18. The int. of $113 for 7 m. 18 d., at 6%.
19. The int. of $150 for 1 y. 3 m., at 6%.
20. The int. of $265 for 1 y. 7 m., at 6%.
21. The amt. of $500 for 2 y., at 8%.
22. The amt. of $763.25 for 1 y. 9 m. 27 d., at 5%.
23. The int. of $1500 for 3 years, at 8%.
24. The int. of $2678 for 1 y. 7 m. 19 d., at 7%.
25. The int. of $2750 for 33 days, at 6%.
26. The int. of $3700 for 63 days, at 7%.
27. The int. of $2500.73 for 93 days, at 5%.
28. What is the int. on a note of $500, from March 10th, 1872, to July 25th, 1872, at 6%?

ANALYSIS.—The time from March 10th to July 10th, is 4 m.; from July 10th to July 25th, it is 15 d. Now, $500 × .0225 = $11.25, *Ans.*

29. What is the amt. of $1250, from July 20th, 1873, to Dec. 29th, 1873, at 6%?

30. What is the amt. of a note of $2000, bearing int. from March 1st, 1872, to Jan. 25th, 1873, at 7%?

31. What is the interest of $4500 for five years, at 6%? The amount?

ANSWERS.

ADDITION.

Ex.	Ans.
	Page 21.
2.	24
3.	25
4.	32
5.	27
6.	26
7.	27
8.	34
9.	37
	Page 22.
1.	45
2.	45
3.	52
4	59
5.	60
6.	61
7.	58
8.	66
	Page 23.
1.	47
2.	41
3.	44
4.	55
5.	53
6.	62
7.	64
8.	70
9.	22 pounds
10.	35 yards
	Page 24.
2.	67
3.	88
4.	79
5.	98
6.	97
7.	887
8.	888
9.	999
	Page 25.
10.	67
11.	77
12.	89
13.	95
14.	88
15.	99
	Page 28.
2.	13142
3.	16424
4.	16189
5.	17140
6.	17374
1.	1714 yds.
2.	2453 lbs.
3.	2359 rods.
4.	$2263
5.	2454 A.
	Page 29.
6.	3280
7.	1936
8.	1232
9.	2093
10.	2377
11.	861 pages
12.	2764 days
13.	$369
14.	$119
15.	939 sheep
16.	1682 sheep
17.	27514
18.	$4018
19.	1799 A.D.
20.	$7835
21.	1886 A.D.
	Page 30.
22.	2387 yds.
23.	$2123
24.	318 years
25.	208 mar.
26.	$2526
27.	45913 men
28.	$58020
29.	1492 days
30.	705 miles
31.	998 acres
32.	11252 years
33.	2413 oz.
34.	2969 lbs.
35.	$1700
36.	1003 A.
	Page 31.
37.	$503
38.	$593
39.	$6674
40.	$53.68
41.	$48.57
42.	$62.55
43.	$539.03
44.	$829.03
45.	$648.62
46.	$714.57
47.	$6366.10

SUBTRACTION.

Ex.	Ans.
	Page 38.
2.	213
3.	324
4.	320
5.	2232
6.	4063
	Page 39.
1.	223 lbs.
2.	263 yds.
3.	4316 hats
4.	$1111
7.	3562 in.
8.	5101 oz.
9.	3000 weeks
10.	3211
11.	4000
	Page 42.
1.	Given
2.	2191
3.	2313
4.	2223
5.	1409
6.	1804
1.	235
2.	3108
3.	3191
4.	144
5.	1174
6.	5218
7.	3101
8.	2228
9.	2167 qts.
10.	144717 bar.
	Page 43.
11.	738 bu.
12.	$723
13.	$405
14.	990
15.	990
16.	7272
17.	3240230
18.	1765509
19.	3929992
20.	6706495
21.	2235109
22.	1000004
23.	582 sh.
24.	$422
25.	83 years
26.	$588
27.	64 years
28.	1722
29.	$930
30.	156 years
	Page 44.
1.	504
2.	100
3.	1409
4.	2640
5.	15164
6.	4325

MULTIPLICATION.

Ex.	Ans.
	Page 50.
2.	84828
3.	66963
4.	48848
5.	55555
6.	606606
7.	996092
8.	770707
9.	888888
	Page 52.
2.	1825 days
3.	$4500
4.	7000 A.
5.	4272
6.	22035
7.	114144
8.	325215
9.	4207380
10.	4450096
11.	6643240
12.	8757720
	Page 54.
2.	93100
3.	139502
4.	247388
5.	344109
6.	681252
	Page 55.
2.	574476
3.	1086912
4.	1830048
5.	3299541
6.	99764360
7.	126257940
8.	296989550
9.	1437399648

Ex.	Ans.
10.	8760 hrs.
11.	48000 rods
12.	$23055
13.	$24250
14.	5393058
15.	18305988
16.	143225262
17.	168465500
18.	213882848
19.	229152462
20.	186691875
21.	411290946
	Page 56.
22.	$55250
23.	22770 miles
24.	16884 yards
25.	$160000
26.	$19250
27.	$40250
28.	$123375
29.	$282750
30.	$200000
31.	$655500
32.	116300 p.
33.	$59985
34.	$188040
	Page 57.
1–3.	Given
4.	36100
5.	45300
6.	2045000
7.	46208000
8.	58241000
9.	3260720000
10.	400728900000
11.	$510
12.	$26500
13.	$20500
14.	63000 cts., or $630
	Page 58.
15–18.	Given
19.	1147000
20.	3399500
21.	85442000
22.	24139500
23.	195500000
24.	21375000000
25.	4186100000
26.	480480120000
27.	$14000
28.	1260000 cts., or $12600
29.	36000 miles
30.	150000 bu.
31.	62500000
32.	3920000000
33.	1412019000
34.	2886000000
	Page 59.
1.	5130 mo.
2.	56250 d.
3.	$576
4.	$460
5.	$1464

SHORT DIVISION.

	Page 65.
2.	2131
3.	2031
4.	2101
5.	10101
6.	2021
7.	1101
8.	1010
9.	10101
	Page 67.
2.	3142
3.	20143
4.	$3121\frac{2}{6}$
5.	$12011\frac{2}{7}$
	Page 68.
2.	11708
3.	$11389\frac{2}{3}$
4.	12004
5.	18062
	Page 69.
6.	$11236\frac{3}{6}$
7.	$10715\frac{3}{7}$
8.	$11202\frac{3}{8}$
9.	$10338\frac{6}{9}$
10.	282 barrels
11.	460 hats
12.	587 barrels
13.	$52\frac{1}{4}$ weeks
14.	125 hrs.
15.	52 boats
16.	24 weeks
17.	70 barrels
18.	170 boxes
19.	$6253
20.	2808 boxes
21.	2203 A.
22.	$10603
23.	2500 hours
24.	125 stages

LONG DIVISION.

Ex.	Ans.
Page 71.	
1.	Given
2.	Given
3.	$3476\frac{2}{5}$
4.	$7275\frac{2}{6}$
Page 72.	
1.	$81\frac{19}{24}$
2.	$2242\frac{13}{18}$
3.	$2175\frac{8}{21}$
4.	$17568\frac{17}{35}$
5.	$12724\frac{33}{47}$
6.	$1610\frac{15}{53}$
7.	$13331\frac{22}{68}$
8.	13379
9.	$10328\frac{52}{84}$
10.	$99289\frac{81}{92}$
11.	204 cows
12.	$200\frac{11}{28}$ A.
13.	36 months
14.	180 stoves
15.	$157\frac{38}{52}$ years
16.	$83\frac{31}{63}$ months
17.	$99\frac{1}{7}$ hhd.
18.	75 yoke
Page 73.	
19.	Given
20.	$13291\frac{222}{657}$
21.	$10266\frac{402}{950}$
22.	$3010\frac{3325}{3435}$
23.	$3022\frac{4000}{7625}$
24.	60 days
25.	$\$68\frac{180}{365}$
26.	$28
Page 74.	
1.	Given
2.	85, 64 rem.
3.	46, 531 rem.
4.	48
5.	437, 5681 rem.
6.	3, 9467 rem.
7.	2, 72364 rem.
8.	10
9.	85, 325764 r.
10.	Given
11.	426, 14 rem.
12.	411, 15 rem.
13.	85, 1545 rem.
14.	78, 281 rem.
15.	31, 342 rem.
16.	8, 16168 rem.
Page 75.	
1.	244
2.	775
Page 76.	
3.	1089
4.	2510
5.	606
6.	1747
7.	6500
Page 77.	
1.	5 days
2.	90000 cts., or $900
3.	$1605
4.	$5
5.	$105\frac{9}{11}$ tons
6.	$945
Page 78.	
7.	685 barrels; $7 a bar.
8.	$2733
9.	$2310
10.	$1305
11.	25435
12.	29379
13.	41884
14.	30 days
15.	4 years
16.	164 cts., 2d 311 cts., 3d
17.	25 days
Page 81.	
1.	Given
2.	$2\times3\times7$
3.	$2\times2\times2\times2\times3$
4.	$2\times2\times3\times5$
5.	$2\times2\times2\times3\times3$
6.	$2\times2\times5\times5$
7.	$5\times5\times5$
8.	$2\times2\times3\times11$
9.	$5\times5\times7$
10.	$2\times2\times2\times5\times5$
11.	$2\times2\times2\times2\times2\times2\times2\times2$
12.	$5\times5\times13$

Ex.	Ans.
13.	$2\times5\times5\times3\times3$
14.	$5\times5\times5\times5$
15.	$2\times2\times2\times5\times5\times5$
16.	$2\times2\times2\times2\times2\times2\times3\times3\times3$
17.	$2\times2\times2\times2\times3\times3\times13$

Page 82.

1. Given
2. Given
3. Given
4. Given
5. $\frac{7}{3}$, or $2\frac{1}{3}$
6. $\frac{11}{8}$, or $1\frac{3}{8}$
7. $\frac{23}{7}$, or $3\frac{2}{7}$
8. 4
9. 14
10. 11
11. $\frac{25}{2}$, or $12\frac{1}{2}$ yd.
12. 36 barrels
13. 44 days.

Page 85.

1. Given
2. 9
3. 16
4. 15
5. 24
6. 15
7. 27
8. 4
9. 256
10. 12
11. 15
12. 15
13. 48
14. 32
15. 12 ft.
16. 20 yds.

Page 87.

1. Given
2. 24
3. 48
4. 90
5. 1008
6. 1800
7. 660
8. 156
9. 648
10. 240
11. 168
12. 12960
13. 864

REDUCTION OF FRACTIONS.

Page 95.

1. Given
2. $\frac{24}{30}$
3. $\frac{25}{40}$
4. $\frac{49}{63}$
5. $\frac{25}{35}$
6. $\frac{45}{75}$
7. $\frac{12}{76}$
8. $\frac{33}{135}$
9. $\frac{92}{268}$
10. $\frac{776}{1000}$
11. Given
12. $\frac{2}{4}$
13. $\frac{1}{3}$
14. $\frac{2}{8}$
15. $\frac{3}{6}$
16. $\frac{2}{4}$
17. $\frac{1}{3}$
18. $\frac{3\frac{3}{5}}{9}$
19. $\frac{4}{6}$
20. $\frac{7}{8}$
21. $\frac{9\frac{1}{3}}{12}$

Page 96.

1. Given
2. Given
3. $\frac{2}{3}$
4. $\frac{2}{5}$
5. $\frac{1}{2}$
6. $\frac{5}{9}$
7. $\frac{2}{3}$
8. $\frac{2}{3}$
9. $\frac{7}{12}$
10. $\frac{2}{3}$
11. $\frac{21}{8}$
12. $\frac{13}{30}$
13. $\frac{2}{3}$
14. $\frac{9}{13}$

Ex. Ans.

15. $\frac{37}{93}$
16. $\frac{61}{103}$
17. $\frac{1}{3}$
18. $\frac{16}{25}$
19. $\frac{1}{3}$
20. $\frac{1}{4}$
21. $\frac{13}{27}$
22. $\frac{71}{103}$
23. $\frac{5}{12}$
24. $\frac{1}{3}$
25. $\frac{1}{3}$
26. $\frac{15}{41}$

Page 97.

1. Given
2. 19
3. $14\frac{2}{3}$
4. $16\frac{1}{4}$
5. $16\frac{1}{5}$
6. 12
7. $14\frac{2}{7}$
8. $12\frac{1}{10}$
9. 15
10. 8
11. $5\frac{10}{23}$
12. $5\frac{3}{4}$
13. $6\frac{2}{3}$
14. $7\frac{5}{9}$
15. $7\frac{23}{24}$
16. 8
17. 12
18. $2\frac{1}{2}$ sh.

Ex. Ans.

19. \$$3\frac{3}{4}$

Page 99.

1. Given
2. Given
3. $\frac{75}{3}$
4. $\frac{215}{5}$
5. $\frac{124}{4}$
6. $\frac{455}{7}$
7. $\frac{57}{5}$
8. $\frac{44}{3}$
9. $\frac{71}{4}$
10. $\frac{156}{5}$
11. $\frac{265}{6}$
12. $\frac{396}{7}$
13. $\frac{547}{8}$
14. $\frac{893}{10}$
15. $\frac{883}{12}$
16. $\frac{1457}{15}$
17. $\frac{2003}{20}$
18. $\frac{2927}{25}$
19. $\frac{6128}{25}$
20. $\frac{21501}{50}$
21. $\frac{50721}{100}$
22. $\frac{7003}{7}$
23. $\frac{8519}{8}$
24. $\frac{8831}{4}$
25. $\frac{15233}{5}$
26. $\frac{52317}{10}$
27. 103 beg.

1. Given

Ex. Ans.

2. Given
3. $\frac{4}{15}$

Page 100.

4. Given
5. Given
6. $\frac{1}{14}$
7. $\frac{35}{144}$
8. $\frac{1}{7}$
9. $\frac{1}{9}$
10. $\frac{5}{36}$
11. $\frac{2}{5}$
12. $\frac{3}{2}$
13. $\frac{3}{44}$
14. $\frac{10}{27}$
15. $\frac{3}{16}$
16. $\frac{88}{35}$
17. $\frac{15}{16}$
18. $\frac{5}{1}$
19. $\frac{3}{2}$
20. $\frac{71}{10}$

Page 101.

1. Given
2. $\frac{24}{30}$
3. $\frac{25}{40}$
4. $\frac{15}{35}$
5. $\frac{21}{72}$
6. $\frac{51}{99}$
7. $\frac{81}{144}$
8. $\frac{87}{171}$
9. $\frac{225}{276}$
10. $\frac{500}{1000}$

Ex.	Ans.
Page 102.	
1.	Given
2.	$\frac{40}{60}, \frac{45}{60}, \frac{12}{60}$
3.	$\frac{8}{12}, \frac{9}{12}$
4.	$\frac{6}{15}, \frac{5}{15}$
5.	$\frac{21}{28}, \frac{12}{28}$
6.	$\frac{36}{60}, \frac{40}{60}, \frac{45}{60}$
7.	$\frac{10}{70}, \frac{35}{70}, \frac{56}{70}$

8. $\frac{48}{216}, \frac{144}{216}, \frac{189}{216}$
9. $\frac{315}{693}, \frac{616}{693}, \frac{297}{693}$
10. $\frac{31080}{63640}, \frac{47730}{63640}, \frac{16340}{63640}$
11. $\frac{19000}{37000}, \frac{18500}{37000}, \frac{25900}{37000}$
12. Given
13. $\frac{25}{10}, \frac{6}{10}, \frac{30}{10}$
14. $\frac{20}{84}, \frac{588}{84}, \frac{441}{84}$

Page 103.

1, 2. Given
3. $\frac{40}{60}, \frac{45}{60}, \frac{12}{60}$
4. $\frac{8}{20}, \frac{5}{20}, \frac{6}{20}$
5. $\frac{21}{24}, \frac{10}{24}, \frac{4}{24}$
6. $\frac{30}{70}, \frac{28}{70}, \frac{49}{70}$
7. $\frac{14}{36}, \frac{20}{36}, \frac{3}{36}$
8. $\frac{168}{105}, \frac{90}{105}, \frac{70}{105}$
9. $\frac{3}{15}, \frac{3}{15}, \frac{5}{15}$
10. $\frac{9}{42}, \frac{28}{42}, \frac{18}{42}, \frac{7}{42}$
11. $\frac{1320}{3465}, \frac{1617}{3465}, \frac{525}{3465}, \frac{2695}{3465}$
12. Given
13. $\frac{16}{40}, \frac{210}{40}, \frac{25}{40}$
14. $\frac{4}{8}, \frac{36}{8}, \frac{5}{8}, \frac{32}{8}$

ADDITION OF FRACTIONS.

Page 105.

1. Given
2. $\frac{21}{8} = 2\frac{5}{8}$ A.
3. $\frac{32}{16} = 2$
4. $\frac{53}{20} = 2\frac{13}{20}$
5. $\frac{53}{25} = 2\frac{3}{25}$
6. $\frac{59}{30} = 1\frac{29}{30}$

Page 106.

3. $\frac{77}{40} = 1\frac{37}{40}$ lb.
4. $\frac{27}{12} = 2\frac{1}{4}$
5. $\frac{103}{70} = 1\frac{33}{70}$
6. $\frac{41}{33} = 1\frac{8}{33}$
7. $\frac{140}{60} = 2\frac{1}{3}$
8. $\frac{79}{40} = 1\frac{39}{40}$
9. $\frac{135}{80} = 1\frac{11}{16}$
10. $\frac{106}{50} = 2\frac{3}{25}$
11. $\frac{305}{144} = 2\frac{17}{144}$
13. $101\frac{5}{6}$ lbs.
14. $52\frac{1}{12}$ yd.
15. $77\frac{13}{20}$ m.
16. \$$24\frac{7}{8}$
17. \$$71\frac{17}{24}$
18. Given
19. $5\frac{13}{15}$
20. $28\frac{7}{9}$
21. $23\frac{1}{6}$
22. $46\frac{11}{12}$

SUBTRACTION OF FRACTIONS.

Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 107.	3. $\frac{5}{16}$	***Page 109.***
1, 2. Given	4. $\frac{3}{40}$	15. Given
3. $\frac{12}{40}=\frac{3}{10}$ T.	5. $\frac{7}{24}$	16. $21\frac{3}{8}$ bu.
4. $\frac{17}{63}$ bu.	6. $\frac{27}{152}$	17. Given
5. $\frac{60}{263}$	7. $\frac{55}{144}$	18. $35\frac{7}{8}$ gal.
6. $\frac{225}{1219}$	8. $\frac{7}{30}$	19. $18\frac{3}{4}$
7. $\frac{89}{1500}$	9. $\frac{24}{72}=\frac{1}{3}$	20. $35\frac{5}{12}$
8. $\frac{89}{1873}$	10. $\frac{17}{80}$	21. $20\frac{5}{6}$
	11. $\frac{44}{75}$	22. $29\frac{1}{8}$
Page 108.	12. $\frac{1}{30}$	24. $\frac{4}{10}=\frac{2}{5}$
1. Given	13. $\frac{28}{225}$	25. $\frac{2}{12}=\frac{1}{6}$
2. $\$\frac{1}{8}$	14. $\frac{11}{240}$	

MULTIPLICATION OF FRACTIONS.

Page 111.

1. Given
2. $\$6\frac{3}{8}$
3. $\$10\frac{5}{12}$
4. $\$150\frac{9}{16}$
5. $12\frac{9}{13}$
6. $15\frac{15}{16}$
7. $8\frac{11}{20}$
8. $7\frac{3}{31}$
9. $6\frac{24}{35}$
10. $12\frac{9}{22}$
11. $26\frac{1}{4}$.
12. $36\frac{62}{83}$
13. $62\frac{1}{4}$
14. $42\frac{31}{32}$
15. Given
16. $\$2.87\frac{1}{2}$
17. $\$843\frac{3}{4}$
18. $781\frac{1}{4}$
19. 1575
20. $2478\frac{1}{8}$
21. $3562\frac{1}{2}$
22. 5000
23. 8775
24. \$654
25. \$750
26. $\$2224\frac{1}{4}$

Page 113.

1. Given
2. 45 cts.
3. $\$62\frac{1}{2}$
4. $33\frac{3}{4}$
5. $48\frac{4}{5}$
6. $22\frac{2}{7}$
7. $32\frac{5}{8}$
8. 30
9. $74\frac{3}{8}$
10. $257\frac{1}{25}$
11. 135
12. 292 days

Page 114.

13. Given
14. \$7
15. 486 miles
16. 325
17. $441\frac{3}{4}$

Ex. Ans.	Ex. Ans.	Ex. Ans.

18. 663
19. $1090\frac{4}{5}$
20. $4161\frac{3}{7}$
21. 6250

Page 115.

1. Given
2. $\$\frac{1}{12}$
3. $\$\frac{3}{20}$
4. $\frac{3}{5}$
5. $\frac{1}{3}$
6. $\frac{1}{6}$
7. $\frac{21}{55}$
8. $\frac{4}{9}$
9. $\frac{19}{111}$
10. $\frac{5}{14}$
11. $\frac{7}{36}$
12. $\frac{1}{15}$
13. $\frac{1}{54}$
14. $\frac{1}{14}$
15. $\frac{2}{15}$
16. Given

1. $\frac{8}{105}$
2. $\frac{1}{4}$
3. $\frac{3}{7}$
4. $11\frac{47}{98}$
5. $50\frac{5}{8}$
6. $14\frac{2}{3}$

Page 116.

7. $\$1.65\frac{3}{8}$
8. $240\frac{7}{8}$
9. $159\frac{9}{10}$
10. $585\frac{15}{16}$
11. 625
12. $1431\frac{1}{4}$
13. $2503\frac{19}{27}$
14. $6737\frac{10}{33}$

DIVISION OF FRACTIONS.

Page 117.

1, 2. Given
3. $\frac{5}{13}$
4. $\frac{11}{36}$
5. $\frac{5}{38}$
6. $\frac{3}{34}$
7. $\frac{9}{116}$
8. $\frac{78}{1333}$
9. $\frac{2}{81}$
10. $\frac{127}{6080}$
11. $\frac{25}{952}$
12. $\frac{103}{7000}$

Page 118.

14. $\$7\frac{1}{2}$
15. $5\frac{2}{9}$
16. $3\frac{7}{10}$
17. $3\frac{35}{44}$
18. $2\frac{293}{345}$
19. $2\frac{293}{560}$
20. $4\frac{321}{625}$

Page 119.

1, 2. Given
3. $53\frac{1}{3}$
4. 88
5. $113\frac{1}{3}$
6. $192\frac{6}{7}$
7. $5\frac{3}{25}$
8. $4\frac{46}{51}$
9. $6\frac{2}{3}$
10. $8\frac{2}{5}$
11. $\$4$
12. 4 miles

Page 120.

2. 6
3. $4\frac{1}{6}$
4. $8\frac{3}{5}$
5. $4\frac{1}{2}$
6. $4\frac{10}{13}$
7. $4\frac{3}{8}$

Page 121.

2. $1\frac{1}{5}$ lb.
3. $2\frac{11}{12}$ pines
4. $6\frac{2}{3}$ lb.

Page 122.

5. $3\frac{1}{3}$
6. $1\frac{4}{5}$
7. $1\frac{1}{14}$

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
8.	$2\frac{4}{5}$	18.	$1\frac{17}{27}$		***Page 123.***
9.	$1\frac{49}{55}$	19.	$2\frac{26}{75}$	4.	$\frac{3}{40}$
10.	5	20.	$2\frac{13}{33}$	5.	$1\frac{1}{21}$
11.	$2\frac{43}{105}$	21.	$3\frac{17}{31}$	6.	$\frac{3}{4}$
12.	$2\frac{53}{68}$	22.	$2\frac{33}{76}$	7.	$\frac{7}{16}$
13.	$2\frac{1}{2}$	23.	$2\frac{949}{1650}$	8.	$\frac{5}{72}$
14.	$2\frac{26}{155}$	25.	5	9.	$\frac{3}{980}$
15.	$2\frac{1}{4}$	26.	$\frac{8}{21}$	10.	$\frac{37}{40}$
16.	$1\frac{15799}{22825}$	27.	60	11.	$\frac{32}{105}$

QUESTIONS FOR REVIEW.

Page 124.

1. $\$35\frac{3}{4}$
2. $\$12\frac{3}{4}$
3. $\$\frac{1}{2}$
4. $\$2\frac{5}{8}$
5. $\$38\frac{19}{32}$
6. $37\frac{1}{2}$ lb.
7. 96 p. k.
8. $146\frac{7}{8}$ lb.
9. $4\frac{1}{2}$ lb.
10. 8 cords
11. 5 bar.
12. $\frac{19}{20}$, sum;
    $\frac{11}{20}$, dif.;
    $\frac{3}{20}$, prod.;
    $3\frac{3}{4}$, quot.
13. $21\frac{3}{7}$ days
14. 8 bales
15. $\$48\frac{1}{4}$

Page 125.

16. $\$9\frac{3}{4}$
17. $4\frac{1}{2}$ lb.
18. $3\frac{2}{3}$
19. $\$17\frac{3}{5}$
20. 3500 lb.
21. $\frac{33}{64}$ sold;
    $\frac{31}{64}$ left
22. 45 weeks
23. $\$95\frac{3}{4}$
24. $\$9\frac{9}{10}$
25. $\frac{23}{96}$ yd.
26. $\frac{7}{16}$
27. 35
28. $1\frac{11}{16}$
29. $6\frac{2}{9}$
30. \$8, flour
31. 14 oranges
32. 5 lb.

FRACTIONAL RELATION OF NUMBERS.

Page 126.

2. $\frac{12}{48}=\frac{1}{4}$;
   $\frac{28}{63}=\frac{4}{9}$
3. $\frac{27}{81}=\frac{1}{3}$;
   $\frac{63}{90}=\frac{7}{10}$;
   $\frac{40}{100}=\frac{2}{5}$

Page 127.

4. $\frac{19}{35}$
5. $\frac{1}{15}$ bu.;
   $\frac{2}{5}$ bu.
6. $\frac{7}{9}$
7. $\frac{10}{13}$
8. Given
9. 24 cts.
10. $\$1093\frac{3}{4}$
11. \$264

Ex. Ans.	Ex. Ans.	Ex. Ans.
12. Given	18. $10\frac{15}{16}$ bu.	5. 180
13. $\frac{3}{103}$		6. 261
14. $\frac{5}{208}$	*Page 128.*	7. $460\frac{10}{19}$
15. $\frac{1}{50}$	2. $42\frac{2}{3}$	8. \$40
16. $\frac{8}{945}$	3. $56\frac{1}{4}$	9. \$$160\frac{5}{7}$
17. \$5	4. $86\frac{2}{5}$	10. 84 yrs.

DECIMAL FRACTIONS.

Page 132.

1. .12
2. .25
3. .05
   .49
5. .119
6. .027
7. .009
8. .013
9. .1345
10. .0236
11. .0039
12. .00007
13. .06 ; .041 ; .007
14. .0201 ; .00752

REDUCTION OF DECIMALS.

Page 133.

1. Given
3. $\frac{24}{100}=\frac{6}{25}$
4. $\frac{135}{1000}=\frac{27}{200}$
5. $\frac{404}{1000}=\frac{101}{250}$
6. $\frac{675}{1000}=\frac{27}{40}$
7. $\frac{1}{25}$
8. $\frac{1}{40}$
9. $\frac{51}{250}$
10. $\frac{41}{400}$
11. $\frac{252}{625}$
12. $\frac{1}{2000}$
13. $\frac{53}{5000}$
14. $\frac{1881}{2500}$
15. $\frac{9}{20000}$
16. $\frac{41}{12500}$
17. $\frac{129}{12500}$
18. $\frac{1929}{15625}$

Page 134.

1. Given
2. .5
3. .2
4. .75
5. .3333+
6. .5
7. .375
8. .4
9. .4166+
10. .9
11. .75
12. .3125
13. .2
14. .0625
15. .025
16. .1
17. .2

ADDITION OF DECIMALS.

Page 136.

1. Given
2. 33.079
3. 16.027
4. 274.251
5. 6.6516
6. 31.465
7. 45.66 yd.
8. \$74.375
9. 72.946 A.
10. 109.841 g.
11. 176.15 r.

SUBTRACTION OF DECIMALS.

Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 137.	7. 3.782	13. 0.045
2. 3.262	8. 99.162	14. 0.0054
3. 6.1682	9. 214.25	15. 0.00009
4. 27.3797	10. 7.3992	16. 6.25 yds.
5. 0.76442	11. 14.993	17. 0.45 ship
6. 49.525	12. 0.6306	18. 62.3 A.

MULTIPLICATION OF DECIMALS.

Page 139.	10. 0.3159	20. 0.000804
1. Given	11. 117.351	21. 111.375
2. Given	12. 18.25	22. 24.375
3. Given	13. 0.114015	23. 393.75
4. Given	14. 8.09792	
5. 0.00381	15. 50.06223	*Page 140.*
6. 0.0363	16. 0.0060024	24. 10.125
7. 0.001058	17. 53.7758	25. 0.00804
8. 760.2128	18. 70	26. 0.00007
9. 25.664	19. 3	27. 0.00035

DIVISION OF DECIMALS.

Page 142.	12. 76	24. 0.0001
1. Given	13. 2.0454+	25. 0.75
2. Given	14. 0.4885+	26. $2.5
3. Given	15. 0.015	27. 4.8 d.
4. Given	16. 3.65	28. $0.5
5. 11 lbs.	17. 0.0385	29. $10.5
6. 51 lots	18. 0.5	30. 561.7+r.
7. 4.312	19. 0.39104	31. 181.05+A.
8. 0.07312	20. 2.9029+	32. 74 times
9. 0.0002806	21. 1000	33. 1.066+ t.
10. 0.0734201	22. 100	34. 4.2857+t.
11. 142.5	23. 0.01	

ADDITION OF U. S. MONEY.

Ex. Ans.

Page 148.

1, 2. Given
3. $1026.692
4. $1631.03
5. $2274.52
6. $284.37
7. $769.73
8. $21.00
9. $284.375
10. $557.43
11. $165.846
12. $265.525
13. $1390.758
14. $1967.06
15. $3071.58
16. $156.13
17. $73.50

SUBTRACTION OF U. S. MONEY.

Page 149.

2. $19.585
3. $15.085
4. $48.918
5. $99.125

Page 150.

6. $0.875
7. $20.625
8. $1.25
9. $187.375
10. $83.58
11. $990.00
12. $160.065
13. $94.86
14. $296.967
15. $19.705

MULTIPLICATION OF U. S. MONEY.

Page 151.

3. $432.85
4. $9.01125
5. $650.052
6. $61987.50
7. $22.50
8. $494.00
9. $28.1875
10. $753.75
11. $13500
12. $1038
13. $31262.50

Page 152.

14. $756.00
15. $19.875
16. $78.80
17. $186.20 s.;
$3.80 dif.

DIVISION OF U. S. MONEY.

Page 153.

7. 11.308+ t.
8. 12 times
9. 63 melons
10. 85.5 lb.
11. $2.125
12. 12½ cts.
13. 14 yearl's
14. 80 A.
15. $5.375
16. $35.63
17. $64.03
18. $2.35
19. 25 cts.
20. $9.625

Page 154.

1. $61.895
2. $1.75 pro.
3. $9.125 dif.
4. $38.31 dif.
5. $25.84
6. $534.60
7. 17 cts.
8. $7.64⅓

Page 155.

9. $10.788+
10. $46.50
11. $67.85
12. 71 horses
13. $1513.125 s.;
$377.875 d.
13. 12.5 tubs
15. $6
16. 500 pair

APPLICATIONS OF U. S. MONEY.

Ex.	Ans.

Page 156.

1. $9.59 amt.
2. $35.25 + $3.19 + $5.04 + $18 + $21 = $82.48, amt.
3. $19.44 + $16.50 + $8.80 + $7.20 = $51.94, debits;
   $9.00 + $8.40 + $8.25 + $18.70 = $44.35, credits.

Page 157.

4. $1.56 + $28.50 + $4.08 + $4.32 + $10.50 = $48.96, amt.
5. $16.70 + $15 + $9 + $10.52 = $51.22, debits;
   $24.00 + $36 + $62 + $26.25 = $148.25, credits;
   Balance due J. Barker, = $97.03
6. $47.34 + $24.50 + $28.50 + $9.75 + $11.10 + $7.50 + $10.80 = $139.44 amt.

REDUCTION.

Page 174.

3. 8714 far.
4. 835 far.
5. 10368d.
6. 110400 far.

Page 175.

9. £6, 1s. 9d.
10. £28, 3s. 2d. 3 f.
11. Given
12. £3, 8s. 9d.
13. 1780 pwt.
14. 61113 gr.
15. 6 lb. 6 oz. 1 pwt.
16. 1 lb. 1 oz. 13 p. 23 gr.
17. 1 oz. 18 pwt.
18. $78
19. 1 cwt. 41 lb. 9 oz.
20. 483704 oz.
21. 3201204 oz.
22. $5.04
23. $656.25
24. 480 dr.
25. 2112 sc.
26. 1 lb. 7 oz. 4 dr. 1 sc.
27. 3 oz. 2 dr. 18 g.
28. 742½ ft.
29. 63532 ft.
30. 46422 yd.

Page 176.

31. Given
32. 94 r. 9 ft.
33. 2 m. 43 r. 8½ ft.
34. 171049 ft.
35. $2.25
36. 1280 rods
37. 10560 st.
38. 5 quarters
39. 6 eighths
40. 2416 16ths
41. 118⅞ yd.
42. $3.20
43. $420
44. 43560 sq. ft.
45. 234407¼ sq. ft.
46. 102729 sq. yd.
47. 102400 sq. r.
48. 5 A. 51 sq. r.
49. 15 A. 100 sq. r.
50. 35 s. y. 6 sq. ft. 49 sq. in.

Page 177.

51. $1361.25
52. $10454.40
53. 1940544 cu. in.
54. 2 cu. y. 1 cu. ft. 1325 cu. in.
55. 33 C. 26 cu. ft.

Ex. Ans.

56. 9664 cu. ft.
57. 1943 cu. ft.
58. $486
59. $76.80
60. $84
61. 442 pts.
62. 51 bu. 1 p. 7 q.
63. $19.20
64. $3.90 prof.
65. 136 boxes
66. 3 bu. 1 p. 3 q.
67. 207 gills
68. 770 qt.
69. 57 gal. 1 q.
70. 10 hhd. 10 g.
71. 45 qt.
72. $63

Page 178.

73. $142.80
74. 278220 sec.
75. 3 d. 4 h. 5 m.
76. 77760 m.
77. 525600 m.
78. 5 y. 185 d. 16 h.
79. 604800 t.
80. $294
81. 3258720 t.
82. 11 d. $8\frac{8}{11}$ h.
83. 16,780''
84. 2° 46' 40''
85. 4 s. 17° 55'
86. 15'' in 1 h.; 1° in 4 m.
87. 2640 sh.
88. 25 r. 10 qu. 18 sh.
89. $\$0.005\frac{5}{24}$
90. 5760 crayons
91. $126
92. 864 pens
93. 75 eggs

SURFACES AND SOLIDS.

Page 179.

1. Given
2. 28 sq. ft.
3. 768 sq. in.
4. 90 sq. r.
5. 30 yds.

Page 180.

6. 24 sq. ft.
7. 50 A.
8. 23040 A.
9. 540 sq. yd.
10. 900 brick
11. $3.60

Page 181.

1. Given
2. 64 cu. in.
3. 45 cu. ft.
4. 240 blocks
5. 455 cu. ft.
6. 98 cu. ft.
7. 378 cu. ft.
8. 160 cu. yd.
9. 81 cu. ft.
10. $243
11. $40.50
12. $40.50
13. 240 cu. ft.
14. 90 cu. ft.

REDUCTION OF DENOMINATE FRACTIONS.

Page 182.

1. Given
2. 1 ft. $10\frac{1}{2}$ in.
3. 7s. 6d.
4. 5 d. 6 hr.

Page 183.

6. $\frac{2}{3}$ qt.
7. $\frac{2}{3}$ pwt.
9. 3 qt. .72 pt.
10. 4 d. 9 hr.
11. 3 pk. 4 qt.

Page 184.

13. $\frac{29}{36}$ yd.
14. $\frac{7}{64}$ bu.
15. £$\frac{5}{8}$
16. Given
17. $\frac{3}{1280}$ m.
18. $\frac{3}{160}$ bu.
19. Given
20. 0.38125 m.
21. £$0.33\frac{1}{3}$
22. $0.09\frac{8}{63}$ hhd.

COMPOUND ADDITION.

Ex. Ans.	Ex. Ans.
Page 186.	7. 186 m. 24 r.
2. £26, 2s. 1d. 3 far.	8. 92 bu. 1 pk. 4 qt.
3. 27 lb. 11 oz.	9. 117¼ yd.
4. 22 yd. 8 in.	10. 54 sq. r. 19 sq. yd. 7 sq. ft.
5. 108 bu. 3 pk. 6 qt. 1 pt.	11. 117 A. 48 sq. r.
6. 25 T. 5 cwt. 23 lb.	12. 4 C. 60 cu. ft.

COMPOUND SUBTRACTION.

Page 187.	*Page 188.*
1. Given	7. 64 A. 143 sq. r.
2. £2, 2s. 2d. 3 far.	8. 41 cu. ft.
3. 5 lb. 11 oz. 5 pwt. 5 gr.	9. 2 T. 252 lb.
4. 1 bu. 0 p. 3 qt.	10. 13° 34′ 57″
5. 7 m. 199 r. 1½ y. 1 ft., or 7 m. 199 r. 1 y. 2 ft. 6 in.	11. 15° 33′ 30″
6. 27 gal. 1 qt.	13. 26 yr. 6 mo. 3 d.
	14. 1 yr. 11 mo. 12 d.
	15. 4 yr. 2 mo. 24 d.
	16. 3 y. 10 m. 23 d.

COMPOUND MULTIPLICATION.

Page 190.		
	6. 61 A. 92 sq. r.	9. 9 T. 625 lb.
4. 184 g. 1 q. 1 p.	7. 12 C. 94 c. ft.	10. £70, 4s. 4d.
5. 256 yd. 2 qr.	8. 67 hr.	11. 85 bu. 1 pk.

COMPOUND DIVISION.

Page 192.		
	6. 9	11. 4400 rails
3. 3 A. 47 sq. r. 7 sq. ft.	7. 6 bu. 2$\frac{6}{7}$ pk.	12. 6 bu. 1$\frac{2}{3}$ pk.
4. 84 cu. ft. 313$\frac{3}{5}$ cu. in.	8. 10 ft. 4 in.	13. 2 A. 24 sq. r.
5. 7½	9. 5 m. 3 fur.	14. 640 times
	10. 2 oz. 12 pwt. 12 gr.	15. 7 bags
		16. 21$\frac{24}{31}$ bundles

PERCENTAGE.

Ex. Ans.

Page 196.

1, 3. Given.
4. 32 yds.
5. $45.936.
6. 42 bu.
7. 80 rods.
8. 133.2 bar.
9. 408 men.

Page 197.

10, 11. Given.
12. $1680.
13. 600 pupils.
14. $15625.

Page 198.

1, 2. Given.
3. $33\frac{1}{3}$%.
4. 300%.
5. 20%.
6. $11\frac{1}{9}$%.
7. 50%.
8. $33\frac{1}{3}$%.
9. 20%.
10. $16\frac{2}{3}$%.
11. 25%.
12. 25%.
13. 25%.
14. 25%.
15. 50%.
16. $33\frac{1}{3}$%.

Page 200.

1, 2. Given.
3. $62.50.
4. $47.024.
5, 6. Given.
7. $35.

PROFIT AND LOSS.

Page 202.

1, 2. Given.
3. $7 gain.
4. $0.75.
5. $2.125.
6. $105.30.
7. $18.40.
8. $65.
9, 10. Given.
11. $93.60.
12. $336.
13. $2025.
14. $5600.

Page 203.

1. Given.
2. 25%.
3. 50%.
4. 50%
5. $28\frac{4}{7}$%.
6. $13\frac{1}{3}$%
7. $33\frac{1}{3}$%.
8. 25%.
9. $42\frac{6}{7}$%.

INTEREST.

Page 207.

1–14. Given.
15. $5.176.
16. $6.668 int.; $65.418 amt.
17. $2.94.
18. $4.294.
19. $11.25.
20. $25.175.
21. $80.00 int.; $580 amt.
22. $69.646 int.; $832.897 amt.
23. $360.
24. $306.683.
25. $15.125.
26. $45.325.
27. $32.301.
28. Given.
29. 5 m. 9 d. time; $33.125 int.; $1283.125 am.
30. 10 m. 24 d. t.; $126.00 int.; $2126 amt.
31. $1350 int.; $5850 amt.

H B Warren of

Show of Sam

and & ye

Harry Harris

First day of January